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# TWO MODELS OF INFLATION CONTROL

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled TWO MODELS OF INFLATION CONTROL submitted by IRENE D. DARIUS in partial fulfilment of the requirements for the degree of Master of Arts.

DEDICATION

"*For The Dustdevils*"

# ABSTRACT

In recent years, rising inflation and low levels of output growth have plagued western economies. Fiscal and monetary policies to lower inflation often put downward pressure on output, while fiscal and monetary policies to stimulate output raise the inflation rate. Therefore, inflation control programs, believed to raise output and lower inflation, have become a popular government policy in Great Britain, the United States and Canada. While inflation control programs have been observed to increase the level of output and lower the rate of inflation in the long-run, periods of lower levels of output exist in the short-run.

It is the aim of this paper to capture these observed findings in two macro-economic models incorporating a voluntary inflation control program. The basic model to which the controls are being applied has received extensive study. Model 1 assumes the control program affects inflationary expectations only. Model 2 assumes the control program affects both inflationary expectations and the demand for real money.

The method of study employed for both models is one of comparative statics and phase diagrams. Two differential equations are formulated one for the real rate of interest and the other for the nominal rate of interest. From these equations the stability conditions are evaluated and examined. Assuming these conditions are satisfied during the control program, phase diagrams and the short-run time paths are drawn to show the movement of the actual inflation rate, expectations, output, the real interest rate, and the nominal interest rate to their long-run equilibrium position. The analysis of the

long-run equilibrium follows the short-run analysis. Finally, an examination of the post-control period completes the study for both models.

The results of both models are similar. In the short-run, cyclical fluctuations of all the endogenous variables may exist. The rate of inflation does fall temporarily in the short-run, but the level of output also falls. The long-run equilibrium is one of a higher level of output, and lower inflationary expectations, real interest rate and nominal interest rate in both models. In Model 1, the actual rate of inflation is the same as its pre-control equilibrium rate, while in Model 2 the rate of inflation is lowered from its pre-control rate. In both models, the post-control equilibrium position is the same as the pre-control equilibrium position. Therefore, unless the control program remains on indefinitely, the inflation control program can only be a temporary policy to lower the inflation rate and raise the level of output.

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## LIST OF SYMBOLS

### PARAMETERS

$\alpha$ — natural logarithm of the maximum fixed amount of aggregate demand.

$\beta$ — speed of adjustment coefficient of expectations to the difference between the actual and expected rate of inflation.

$\gamma$ — speed of adjustment coefficient of output to the difference between the actual and expected rates of inflation.

$\delta$ — speed of adjustment coefficient of expectations to the difference between the controlled and actual rates of inflation.

$\varepsilon$ — elasticity of the demand for real money balances with respect to the ratio $\frac{P}{P_g}$ .

$\eta$ — natural logarithm of the maximum amount of real balances demanded in Cagan's model.

$\theta$ — sensitivity of the natural logarithm of the real balances demanded to the expected rate of inflation in Cagan's model

$\lambda$ — nominal interest sensitivity of money demand.

$\mu$ — real interest sensitivity of aggregate demand.

$\nu$ — natural level of output.

$\rho_\nu$ — natural, real rate of interest associated with $\nu$ in the IS equation.

| | |
|---|---|
| $\tau$ | coefficient of adjustment in Cagan's money market dis-equilibrium equation. |
| $\phi$ | natural logarithm of the maximum fixed amount of real balances demand. |
| $\psi$ | Frenkel's speed of adjustment coefficient of short-run expectations to the difference between the long-run expected rate of inflation and the actual rate of inflation. |
| $\omega$ | speed of adjustment coefficient of the difference between the actual inflation rate and the expected long-run rate of inflation in Frenkel's model. |

VARIABLES

| | |
|---|---|
| $\pi$ | actual rate of inflation. |
| $\pi^e$ | expected rate of inflation. |
| $\pi_g$ | government controlled rate of inflation. |
| $\pi_n$ | Frenkel's expected long-run rate of inflation. |
| C | aggregate real consumption. |
| I | aggregate real investment. |
| r | nominal rate of interest. |
| M | nominal money supply. |
| $M^d$ | nominal amount of money demanded. |
| $\frac{M}{P}$ | supply of real money balances. |

| | |
|---|---|
| $\frac{\dot{M}}{M}$ | rate of change in the money supply, (government policy variable). |
| $P$ | actual price level. |
| $P_g$ | price corresponding to the government controlled inflation rate. |
| $\rho$ | real rate of interest. |
| $Y$ | nominal output level. |
| $y$ | real output level. |

## OPERATORS

| | |
|---|---|
| $\ell n$ | natural logarithm. |
| $\bullet$ | time derivative, $\frac{d}{dt}$. |

# CHAPTER I

## REVIEW OF THE LITERATURE

Throughout history, periods of high inflation have frequently been observed. Central monetary authorities often use expansionary monetary policies to stimulate the economy to higher levels of employment and output. Monetary policy is useless as a long-run policy if the monetarist belief holds that the ultimate, full effect of a monetary shock is borne by the price level. The rising inflation rate resulting from such a monetary policy may cause the governing body to impose an inflation control program. The effectiveness of the control program depends critically on the formation of inflationary expectations entertained by the public. The formulation of inflationary expectations has proceeded in two main directions. "Rational expectations" assumes people adjust their inflationary expectations directly to changes in the growth rate of the money supply. This method maintains that people have the foresight to see that changes in the money supply growth rate will have no long-run effect on output and employment. The other expectations formation process is "adaptive expectations". This method assumes expectations adjust to past observations of the actual inflation rate. Perfect foresight may also exist with adaptive expectations. The adaptive expectations formulation is used in this thesis because rational expectations would require a stochastic model be employed which is beyond the scope of this paper.

It is the goal of this paper to model an inflation control program which raises the long-run level of output and employment, and lowers the long-run inflation rate.

The literature examined may be divided into three major sections. The first section is the Neoclassical approach. The effects of monetary growth rate changes on the monetary sector are examined. Extended models including the real sector are examined in the second section. These models show the effects, on the real sector, of changing the growth rate of the money supply. The final section refers to models of price controls. The complete models are not examined in this section, only the price control mechanisms utilized. Empirical studies of inflation control programs are not extensively reviewed in this paper as they are numerous and a review of them would constitute a thesis by itself. Nevertheless, the models are constructed to try and capture these observed findings.

## A. THE NEOCLASSICAL APPROACH

The models examined all originate with the work of P. Cagan [6] on hyperinflation. Cagan builds the model to explain the effects of hyperinflation on the demand for real balances. He then conducts an econometric study of inflation in a number of European countries. The first equation of the model is a semi-logarithmic demand function for money with an expected rate of inflation.

$$\ell n \left(\frac{M^d}{P}\right) = \eta - \theta\pi^e \; ; \qquad \theta > 0 \quad . \tag{1.1}$$

The nominal amount of money demand is $M^d$. $P$ is the price level, and $\pi^e$ is the expected rate of inflation. There are two parameters: $\theta$, measuring the sensitivity of the logarithm of real balances demanded to changes in the expected rate of inflation, and $\eta$, the logarithm of the maximum amount of real balances demanded, when $\pi^e = 0$. This

function is based on the belief that people desire to hold a certain amount of real cash balances dependent on their real wealth and real income, represented by $\eta$, and the expected return of their wealth, represented by $\theta\pi^e$. The demand for real cash balances varies inversely with the expected rate of inflation as the cost of holding money is the actual rate of inflation plus the real rate of interest, or the opportunity cost (i.e., the cost of not purchasing durable goods and the cost of not holding securities). Since portfolio decisions are not examined in his model, the real rate of interest is set aside in the demand for money function. As people expect the rate of inflation to rise, they decrease their demand for real cash balances.

The supply of nominal money, $M$, is controlled by the monetary authorities. There is no reason why the supply of nominal money should change to equate the supply of real balances with the demand for real balances. Cagan chooses to examine a disequilibrium model in the money market by an equation of the form:

$$\frac{d}{dt}\left(\ell n \frac{M}{P}\right) = \tau\left[\ell n \left(\frac{M^d}{P}\right) - \ell n \left(\frac{M}{P}\right)\right] \quad . \tag{1.2}$$

The percentage change of the supply of real balances, $\frac{M}{P}$, adjusts linearly to the difference between the logarithm of the demand for real balances and the logarithm of the supply of real balances. The coefficient of adjustment, $\tau$, represents the speed by which the supply of real balances adjusts to the demand for real cash balances. Since government authorities regulate the nominal supply of money, the supply of real money must adjust to the demand for real money through

changes in the price level, P. The price level changes directly with the rate of inflation. The inflation rate rises due to increases in aggregate demand created by the excess supply of real cash balances. Cagan assumes that $\tau$ is large enough during periods of hyperinflation so that the supply of real balances adjusts almost instantaneously to the demand for real balances. This permits him to assume money market equilibrium, $\frac{M}{P} = \frac{M^d}{P}$.

The final equation in Cagan's model is the adaptive expectations equation:

$$\dot{\pi}^e = \beta(\pi - \pi^e) \ ; \qquad \beta > 0 \tag{1.3}$$

where $\beta$ is the speed of adjustment coefficient of expectations to the discrepancy between the actual and expected rates of inflation, and $\dot{\pi}^e$ is the change in the expected rate of inflation. Eq (1.3) expresses expectations as a weighted average of past rates of inflation with the weights declining geometrically backward through time.

Using the assumption of money market equilibrium, the rate of inflation can be solved for in terms of the rate of change in the money supply and the expected rate of inflation. This is accomplished by differentiating the demand for money function, Eq (1.1), with respect to time, and combining it with Eq (1.3). The resulting equation is:

$$\pi = \frac{\dot{M}/M - \theta\beta\pi^e}{1-\theta\beta} \ . \tag{1.4}$$

The stability condition for this equation is:

$$1 - \theta\beta > 0 \ .$$

The stability condition is not satisfied when the speed of adjustment coefficient becomes at least as large as the reciprocal of the inflation expectation sensitivity coefficient of the demand for real balances. When this occurs the inflation rate changes directly with expectations in Eq (1.4). Feeding this new value of $\pi$, derived in Eq (1.4), back into Eq (1.3) causes $\pi^e$ to rise again which in turn causes $\pi$ to increase again. The larger the value of $\beta$, the faster this process occurs as expectations adjust rapidly to changes in the actual rate of inflation. A paradoxical result arises from this model when the stability condition is not satisfied. Increasing the rate of change in the money supply, $\frac{\dot{M}}{M}$, causes the rate of inflation to decline. We can observe this from Eq (1.4) when $\theta\beta \geq 1$.

When the system is stable, $\theta\beta < 1$, an increase in the rate of change in the money supply causes the inflation rate to increase. In the long-run equilibrium position, expectations are realized, $\pi^e = \pi$. Replacing this condition in Eq (1.4) and simplifying it yields the long-run Cambridge Cash-Balance equation result that the rate of inflation equals the rate of change in the money supply, $\pi = \frac{\dot{M}}{M}$. The conclusion is that in the long-run, under stable conditions, increasing the money supply growth rate only results in an equal increase in the inflation rate.

Goldman [18] rectifies the result of an inverse relationship between changes in the growth rate of the money supply and the rate of inflation in an unstable system. To accomplish this, he stipulates that money market disequilibrium may exist. The equation he uses is Cagan's original money market disequilibrium equation, Eq (1.2). By solving

Cagan's system of equations using Eq (1.2) and without assuming money market equilibrium, Goldman eliminates the paradoxical result that a positive monetary shock causes the inflation rate to fall in an unstable system of equations. Whether stability conditions prevail or not, Goldman finds a positive monetary shock causes expectations to rise, thereby increasing inflation. The stability condition is less stringent than Cagan's. Specifically, the condition is:

$$\tau(1-\theta\beta) + \beta > 0 .$$

J. Frenkel [13] sets out to change the effect of monetary policy on inflation in Cagan's model. Through empirical observations, Frenkel notes a monetary shock initially causes real balances to increase and the nominal interest rate to decline. Cagan's model initially causes the actual inflation rate to rise above the new level of the money supply growth rate so real balances do not increase at all. Feeling this is caused by an inappropriate formulation of the expectations process, Frenkel adjusts Cagan's model by adding another term to Eq (1.3). Frenkel maintains Cagan's basic model including the simplified money market equilibrium condition, $M = M^d$ at each moment in time. He argues there exists some expected long-run rate of inflation, $\pi_n$, which the prevailing rate of inflation oscillates about and adjusts towards. The mechanism of adjustment of this long-run expected inflation rate is an adaptive process towards the actual rate of inflation:

$$\dot{\pi}_n = \omega(\pi-\pi_n) \ ; \qquad \omega > 0 \tag{1.5}$$

where $\omega$ is the speed of adjustment coefficient. The short-run expected rate of inflation not only adjusts to the actual rate of inflation as in Cagan's model, but also to the difference between the long-run expected rate and the actual rate:

$$\dot{\pi}^e = \psi(\pi_n - \pi) + \beta(\pi - \pi^e) \; ; \qquad \psi > \beta > 0 \; . \tag{1.6}$$

The condition $\psi > \beta$ is required if the model's behavior is to be consistent with observed short-run behavior. The system is in a steady-state equilibrium position only when $\pi = \pi_n = \pi^e = \frac{\dot{M}}{M}$. The Cambridge Cash-Balance equation result remains the same. Solving the system of equations, Eq (1.1), Eq (1.5), Eq (1.6) and $M = M^d$, for the rate of inflation results in the expression:

$$\pi = \frac{1}{1+\theta(\psi-\beta)} \left(\frac{\dot{M}}{M} + \theta\psi\pi_n - \theta\beta\pi^e\right) \; . \tag{1.7}$$

Initially both expected inflation rates are unaffected by an increase in the growth rate of the money supply as seen by Eq (1.5) and Eq (1.6). The rise of inflation is then seen to be initially less than the rise in the growth rate of the money supply, in Eq (1.7). This result allows the observed initial real balance increase to occur. Eq (1.5) shows that an increase in inflation causes long-run expectations to start rising while the short-run expectations (Eq (1.6)) fall initially as both $\pi_n$ and $\pi^e$ are still at their initial values. Since $\pi_n$ lags behind the rising $\pi$ and dominates the short-run expectations formation process, real cash balances may continue to increase for a short time period. Eventually $\dot{\pi}^e$ becomes positive causing real balances to start declining. The economic interpretation of this is that people

are willing to increase their holdings of nominal cash balances but not as much as the new supply created by the higher money supply growth rate. This forces the inflation rate up, but less than the rise in the growth rate of the money supply. People expect the inflation rate to return to its previous level again, so the actual rate rises slowly allowing real balances to increase. Over time, they begin to realize the inflation rate is not declining to its intial position, so they begin to alter their long-run expectations along with the observed, actual inflation rate. This causes their short-run expectations to climb, and eventually real balances fall.

Upon solving Frenkel's system of equations, one finds the stability condition to be less restrictive than that of Cagan's and almost identical to Goldman's condition cited earlier:

$$\omega(1-\theta\beta) + \beta > 0 \ .$$

Here $\omega$ is associated with long-run expectations while the parameter $\tau$, in Goldman's condition, is associated with money market disequilibrium.

Frenkel [13] formulates a two part expectations process model obtaining results which are consistent with observed real balance effects in the short-run, and in the process, he eliminates the paradox in the unstable case of Cagan's model. Goldman [18] eliminates this paradox by maintaining Cagan's original real balance disequilibrium equation. Neither Frenkel or Goldman consider adding the real sector to arrive at a more complete model.

Yarrow [43] examines the Cagan [6] / Goldman [18] model cited above. He utilizes Cagan's disequilibrium equation, Eq (1.2) and Cagan's expectations equation, Eq (1.3), but leaves the demand for money equation in a general form:

$$\frac{M^d}{P} = g(\pi^e) \qquad \text{where} \qquad g'(\pi^e) < 0 \quad .$$

Running through the same procedure as Goldman [18], he calculates the stability condition for this general model. He then proceeds to calculate the exact stability conditions for three forms of money demand functions: a linear, a semi-logarithmic and a constant elasticity of demand for money function. The second form, the semi-logarithmic demand for money function, is the one that Cagan [6], Frenkel [13], and Goldman [18], use in their models. Only the semi-logarithmic money demand function has a stability condition that is independent of the growth rate of the money supply. The linear form stability condition is not satisfied for large growth rates of money supply. When this occurs, the price level and inflationary expectations rise continuously. This is caused by continual excess supply of real cash balances, creating excess aggregate demand for output, forcing prices higher.

In the constant elasticity money demand function model, small increases in the money supply growth rate may create unstable conditions. This implies that low rates of monetary expansion cause a deflationary period where the price level and expectations fall continuously. This means the demand for real cash balances is increasing faster than the

supply so there is always an excess demand for real money blances. This causes aggregate demand to decline, sending prices down further. In essence, Yarrow [43] develops a generalized Cagan [6]/ Goldman [18] model and provides some insight into the effects employing various demand for money functions have on the stability condition.

## B. THE EXTENDED KEYNESIAN APPROACH

Stability conditions cannot be derived for the aggregate economy from the above models as they ignore the effects of the real sector. Complete equilibrium models incorporating expectations into them have taken three main approaches: the Phillips curve approach, the IS-LM approach, and a combination of the two. The Phillips curve approach proceeds from the aggregate supply side vis-à-vis the labour market, while the IS-LM is an aggregate demand approach. In the preceding Neoclassical models the price level changes to maintain real cash balances at equilibrium. In the extended Keynesian models the price level changes due to excess aggregate demand, or inflationary expectations. This means the inflationary process may be generated from either the monetary or the real sector in extended Keynesian models.

Yarrow [43] in the same article mentioned above, extends his general monetary model into an extended Keynesian model to incorporate the real sector. Real aggregate demand of the private sector is the summation of real consumption and real investment where consumption is a function of real income only and investment depends on the

expected real rate of interest. Solving for real income as an explicit function of the expected real interest rate only, yields the IS function:

$$y = y(r-\pi^e) \; ; \qquad \frac{dy}{d(r-\pi^e)} < 0$$

where $r$ is the nominal rate of interest and $r-\pi^e$ is the expected real interest rate. The assumption imposed on this function, $dy/d(r-\pi^e) < 0$, means that the level of real aggregate demand falls as the expected real rate of interest increases, implying a downward sloping IS curve in $r$-$y$ space. Inflation enters the real sector via an expectations augmented Phillips curve of the form:

$$\pi = f(y) + \pi^e \; ; \qquad \frac{df(y)}{dy} > 0 \quad . \tag{1.8}$$

This says that inflation is an increasing function of the real level of output and increases one-to-one with the expected rate of inflation, $\pi^e$. It is assumed that there exists some positive level of output, $\nu$, for which $f(\nu) = 0$, where expectations are realized, $\pi = \pi^e$.

In the money market, the income elasticity of real balances demanded is assumed to be unitary. Also, the demand for real cash balances is a function of the nominal interest rate and the expected rate of inflation in the following manner:

$$\frac{M^d}{P} = yg(r,\pi^e) \; ; \qquad \frac{\partial g}{\partial r}, \frac{\partial g}{\partial \pi^e} < 0 \; .$$

Yarrow maintains Cagan's money market equilibrium condition and expectations equation, Eq (1.3).

The solution to Yarrow's system in terms of the changes in the nominal interest rate and the expected rate of inflation with respect to time yields the long-run equilibrium result for inflation that $\pi = \frac{\dot{M}}{M}$ . The stability condition is similar to the one Yarrow derives for the monetary sector alone using either the general functional form or the specific money demand functions.

D. Taylor [38] examines the specific semi-logarithmic demand for money case that Yarrow [43] talks about. It is an equilibrium model in that both the product and money markets are assumed to be in equilibrium at each moment in time. The model is formulated, as in Yarrow's, to examine the effects of a once-and-for-all change in the growth rate of the money supply on the real and nominal rates of interest, and the rate of inflation. Both the IS and LM curves are expressed in semi-logarithmic forms. The IS curve is a function of $\ell n\ y$ and the real rate of interest, $\rho$, and the LM curve is a function of $\ell n\ y$ and the nominal rate of interest, $r$.

$$\ell n\ y = \alpha - \mu\rho \qquad \text{product market equilibrium or IS function.} \tag{1.9}$$

$$\ell n\ \left(\frac{M}{Py}\right) = \phi - \lambda r \qquad \text{money market equilibrium or LM function.} \tag{1.10}$$

The two functions are connected by a simplified version of I. Fisher's interest rate equation:

$$r = \rho + \pi^e .$$

Because of the importance of this equation, it is discussed before continuing on with D. Taylor's model.

Much work has centered on Fisher's interest rate equation. M. Friedman [15] explains the time path adjustment of the nominal interest rate when there is an exogenous change in the growth rate of the money supply. Initially the nominal rate of interst falls causing the real rate to drop. This drop creates increased expenditures in consumption and investment, raising output, income, and the demand for loans. These last influences cause the price level to rise. The real rate of interest returns to its natural level and the nominal rate rises above its former equilibrium because in the long-run, creditors demand the same long-run real return on their investment. Friedman [15] cites studies of Brazil, Chile and the U.S. which suggest that a time period of two to three years is required for the nominal interest rate to adjust to its new equilibrium.

W. Gibson [17] undertakes an extensive econometric study of the interest rate structure in the U.S. He concludes that Fisher's equation does hold but not on a one-to-one basis as the equation states. He explains this result by the effect of business cycles on the interest rate. He hypothesizes that business cycles affect the value of the nominal interest rate as people include these cycles in their expectations. This creates cyclical fluctuations in the nominal interest rate of an approximately three years in length. Gibson does find that the change in the real rate of interest is so small that its

effect on the nominal interest rate can be neglected. Therefore, the simplified version of the interest rate equation can be employed.

The last two equations Taylor [38] uses are Cagan's adaptive expectations equation, Eq (1.3), and an equation that is a form of Yarrow's Phillips curve equation, Eq (1.8):

$$\ell n\ y = \ell n\ \nu + \gamma(\pi - \pi^e) \ ,$$

where

$$\pi - \pi^e = (\ell n\ y - \ell n\ \nu)/\gamma = f(y) \ .$$

The constant level of real output is specified by $\nu$. This equation summarizes the effects of both the labour market and the supply of output in the following manner. An exogenous increase in the growth rate of the money supply causes an increase in aggregate demand by raising consumption and investment. This causes the price level and the actual inflation rate to rise. Labour base their wage demands on the expected inflation rate. Since the expected rate, as explained by Eq (1.3), lags behind the actual inflation rate, the real wage rate will fall. Entrepreneurs regard the actual inflation rate when making decisions on the supply of output. Since revenues are increasing faster than costs, profits are rising, resulting in an increase in output above the natural level. Over time, as expectations increase, wage demands rise resulting in a higher real wage rate. This causes costs to rise and profits to fall. Therefore, entrepreneurs decrease the supply of output. Equilibrium is attained when expectations are realized causing output to be at its natural level again. This process relates to the Phillips curve which is summarised into one equation here.(1)

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(1) R. Dornbusch and S. Fischer [9].

Taylor differentiates both the IS and LM functions with respect to time, eliminates output, and substitutes the IS curve into the natural level of output equation. Taylor then solves for the endogenous variables $r$, $\rho$, and $\pi$, in terms of the exogenous variables $\frac{\dot{M}}{M}$, $\ln \nu$, and $\rho_\nu$, where $\rho_\nu$ is the real, natural long-run rate of interest. The real, natural interest rate is the equilibrium rate of interest corresponding to the natural level of output determined by the IS function, $\ln \nu = \alpha - \mu\rho_\nu$. This results in two independent differential equations whose roots determine the dynamic adjustment path to the new equilibrium after a monetary shock, providing the system is stable. The stability condition is identical to Cagan's as is the equilibrium position for the inflation rate. After a permanent increase in the monetary growth rate, the rate of inflation rises to equal the growth rate of the money supply. The nominal rate of interest is equal to the real rate of interest plus the rate of inflation in equilibrium. This is readily seen by Fisher's equation. Thus a change in the monetary growth rate influences all nominal values. All real values, except real money balances, return to their natural levels. This means a monetary shock of this type has only a short-run effect on the level of output, and the real interest rate.

The dynamic adjustment path is one where inflation rises initially but less than the change in the money supply growth rate. This allows both rates of interest to fall and real money balances to increase. Over time the inflation rate overshoots its equilibrium position due to the rising aggregate demand, and then adjusts downward as both interest rates rise to their respective equilibrium positions.

Real output returns to its former equilibrium while real money balances are smaller than their previous equilibrium level. These are the same results Frenkel [13] achieves when he changes the expectations formation process in Cagan's model [6]. Here, by adding the real sector into the analysis and maintaining Cagan's expectations process Taylor develops the same observable results.

J. Vanderkamp [40] approaches the inflation problem from the labour market side. The money market is represented by the Cambridge Cash-Balance equation, and he utilizes a Phillips curve of the same form as that of Yarrow's, Eq (1.8). Instead of using output as Yarrow [43] does, he has inflation as an increasing function of the inverse of the unemployment rate. In place of Cagan's adaptive expectations, Eq (1.3), Vanderkamp employs a function representing the unemployment rate where the rate of change of the unemployment rate adjusts to the difference between the actual growth rate of output and the natural growth rate, which is assumed to be constant. As the growth rate of output exceeds the natural rate, the unemployment rate falls causing the rate of inflation to increase. A change in the growth of the money supply yields the same results as those in Taylor's model [38].

One can find a correspondence between the equations Vanderkamp [40] utilizes and those Taylor [38] uses. The long-run equilibrium positions of inflation are identical in both models. Both models assume money market equilibrium at each point in time. Vanderkamp's Phillips curve analogy employs a natural rate of unemployment corresponding to Taylor's natural

rate of output. The unemployment rate adjustment equation corresponds to Cagan's adaptive expectations equation in Taylor's model.

These models are all monetarist in nature implying that changes in the money supply growth rate do not have any long lasting effects on output, but do have an effect on inflation. This result leads to the conjecture that the natural rate may be changed via institutional or technological changes, or possibly an expansionary fiscal policy in order to increase the level of output and employment. Models which allow for the possibility of money illusion show monetary policy affecting the level of real output, and other real variables. An example of this is B. Scarfe's paper "Optimal Monetary Policy with a Trade-Off Function" [35].

## C. MODELS OF PRICE CONTROLS

Little work has been done in the field of price controls in economic models. The majority of the work in this field has been empirical studies. These studies are not examined here as it would not be feasible, and examining the empirical studies would be a major paper by itself.(2) Therefore, only the price control mechanism of three models are briefly examined here. One of the models, Possen's Model [32] applies a price control mechanism to a Keynesian model. The other two models examined, Howard [21] and Baily [2], are labour

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(2) See Lipsey and Parkin [24], Lipsey [25], Mogil [27], Parkin [31], Reid [33], and Weintraub [42].

market models.

Possen [32] designs a complex price and wage control system in a Keynesian framework. The model is an equilibrium model as he believes controls do not affect the equilibrium position, only the speed of adjustment and the path of approach. Controls are imposed on the assumption that the government has the power to enforce them, and people believe the controls are permanent and will achieve their goal.

The labour market is represented by a Phillips curve. A wage control is imposed on the labour market so that the slope of the Phillips curve is changed. This conforms with evidence of R. Lipsey and J. Parkin [24] in their study of the effects of controls in Britain. Initially, the controls cause expectations to be equated with the nominal rate of wage inflation which is controlled by the government. Over time, as people begin to realize the labour market is in disequilibrium, they take this into account in formulating their wage demands. A similar control setup exists in the product market. Possen's results show that in the long-run, controls increase (decrease) the level of employment and output when expectations are above (below) the actual inflation rate as the control program makes people aware of the actual inflation rate. The rate of inflation falls when the inflation ceiling is set below the actual inflation rate.

By applying controls along with monetary or fiscal policies, Possen concludes that the rate of inflation can be prevented from rising as the level of employment increases to its new equilibrium. This is accomplished by lessening some of the demand pressures in

both the labour and commodity markets.

The other two inflation control programs focus their attention on the labour market. J. Howard [21] hypothesizes a control program where the government issues a type of "income certificate" for each labourer hired. The certificates have a fixed amount of payment that a labourer may receive. By controlling the number of certificates in the labour market, the government can limit the flow of money between households and firms. The only way labourers can receive higher wages is by having more than one certificate but this situation will lower the level of employment. The inflation rate is not totally controlled as there are no constraints on capital costs.

M. Baily [2] proposes a control program similar to Howard's. At the beginning of each period, firms send requests of their desired price and wage levels for the next period to the policy makers. By using all the requisitions the policy makers calculate a price index. Based on the price index, they then accept the requests or set the price and wage levels firms are permitted. These requests become binding contracts, and movements in prices or wages in either direction do not occur.

Baily's theory is not free of problems. Firstly, his theory assumes all firms are price setters and therefore, are capable of submitting a desired price level to the policy makers. Also, to calculate a price index, it is assumed all wage contracts are due at the same time. Finally, the program does not allow for uncertain exogenous changes that may affect the actual inflation rate which firms would react to in an uncontrolled economy.

## D. SUMMARY

Cagan [6] builds a model of the monetary sector which shows the effects of the growth rate of the money supply on inflation. Goldman [18] and Frenkel [13] make adjustments to the model which result in a more realistic short-run time path of inflation after an exogenous change in the money supply growth rate.

Yarrow [43], Taylor [38] and Vanderkamp [40], adjust Cagan's model to include the real sector. The long-run effect of an exogenous change in the money supply growth rate is the same in all three models. The actual rate of inflation equals the growth rate of the money supply. Both Vanderkamp and Taylor examine the short-run effect on output while Taylor alone examines the short-run movement of the real and nominal interest rates.

Possen [32], Howard [21], and Baily [2] examine the effects of price control mechanisms on inflation, output and employment. Possen builds a complex model and imposes controls on both the labour and product markets. Howard and Baily impose a control program on the labour market only. Possen details the short-run movement of output, employment and inflation while the other two articles briefly discuss these effects.

It is the intent of this paper to build a model of an inflation control program and investigate both the short-run and long-run effects on output, inflationary expectations, the nominal interest rate, and the real interest rate. The post-control period is also examined.

# CHAPTER II

## MODEL 1

Throughout their history, industrial nations have been plagued with persistent unemployment and inflation. Aside from monetary and fiscal policy methods, governments have attempted to alleviate the problem of inflation via inflation control programs. It is believed that by lowering the rate of inflation the level of output and employment will rise. As seen in the previous chapter, control programs have not been analyzed in a simple macro-economic model. This chapter investigates the consequences of applying an inflation control program to a simple monetary model.

The basic theoretical development in this chapter is to apply a government program of inflation control to D. Taylor's monetary model [38] in such a way that there is only an indirect influence on inflation, via people's inflationary expectations. The focus of our examination is on the effects the control program has on the endogenous variables: the inflation rate, output, the real rate of interest, the nominal interest rate and the expected inflation rate. For simplicity, it is assumed that the natural growth rate of output is zero.

First order differential equations are derived for both the real and nominal interest rates. The stability conditions for the model are then derived from these two equations. Phase diagrams are drawn and examined for the stable model followed by the time paths towards a steady-state equilibrium position. The time paths are examined for all

five endogenous variables. Often inflation control programs are short in duration. Therefore, an analysis of the post-control period follows the examination of the control period. Both the stable and unstable post-control periods are examined.

The main discussion is directed to the case where the government controlled inflation rate is set below the growth of the money supply. The reason for this is that initially the actual rate of inflation equals the growth rate of the money supply and the government would probably set the controlled rate below the actual rate of inflation. The other two cases, the controlled inflation rate equals the growth rate of the money supply and the controlled inflation rate is greater than the money supply growth rate, are briefly examined.

## A. CONSTRUCTING THE MODEL

Aggregate demand in the private sector is the summation of consumption and investment expenditures. Investment and consumption decisions are formulated on the real rate of interest, $\rho$, as security holders are interested in the real rate of interest on their capital investment. The investment function is expressed by:

$$I = e^{\alpha_1 - \mu_1 \rho} \quad ; \qquad \mu_1 > 0 \quad . \tag{2.1}$$

The constant $e^{\alpha_1}$ is the maximum fixed amount of investment demand, when the real rate of interest is zero. The interest sensitivity parameter is $\mu_1$, and $I$ is the aggregate investment demanded.

Investment demand has an inverse relationship with the real rate of interest which is the cost of financing the investment.

Consumption is a function of both the real rate of interest and the real level of output, $y$:

$$C = y(1-e^{\mu_2\rho-\alpha_2}) \; ; \qquad \mu_2 > 0 \quad . \tag{2.2}$$

The marginal propensity to consume out of income, $\frac{\partial C}{\partial y}$, is between zero and one, for $\alpha_2 > \mu_2\rho$. Consumption is a decreasing function of the real rate of interest:

$$\frac{\partial C}{\partial \rho} = -\mu_2 y e^{\mu_2\rho-\alpha_2} < 0 \quad .$$

This consumption function is consistent with Taylor's model [38]. Product market equilibrium then is given by:

$$\begin{aligned} y &= C + I \\ &= y(1-e^{\mu_2\rho-\alpha_2}) + e^{\alpha_1-\mu_1\rho} \text{, and thus} \\ y &= e^{\alpha_1+\alpha_2-(\mu_1+\mu_2)\rho} \quad . \end{aligned}$$

Therefore, the product market equilibrium equation is:

$$y = e^{\alpha-\mu\rho} \; ; \qquad \text{where} \quad \alpha = \alpha_1+\alpha_2 \tag{2.3}$$

$$\mu = \mu_1+\mu_2 \, , \qquad \mu > 0 \quad .$$

The maximum amount of aggregate demand is $e^{\alpha}$, when $\rho = 0$. The parameter $\mu$ is the real interest rate sensitivity of aggregate demand.

The demand for real money balances is a function of real income and the nominal interest rate. It is assumed that the elasticity of real cash balances demanded with respect to real income is unitary. Real money demanded depends on the nominal interest rate in an exponential manner.

$$\frac{M^d}{P} = ye^{\phi-\lambda r} \quad ; \qquad \lambda > 0 \quad . \tag{2.4}$$

The interest sensitivity of demand for money is $\lambda$, and $\phi$ is a constant parameter such that $ye^{\phi}$ represents the maximum level of the demand for real cash balances, when $r = 0$. The rationale for using the nominal rate of interest in the money demand function is that creditors expect inflation to continue and therefore incorporate the expected inflation rate in their lending rate to maintain their desired rate of return. Debtors are willing to pay the extra amount as they also expect inflationary increases.(3) Equilibrium occurs in the money market when real money balances demanded equal the supply of real money.

It is assumed that both the product and money markets are in equilibrium at each point in time. Rewriting Eq (2.3) and Eq (2.4) in logarithmic forms results in a semi-logarithmic IS-LM model:

$$\ln y = \alpha - \mu\rho \quad ; \qquad \text{IS function} \tag{2.5}$$

$$\ln\left(\frac{M}{Py}\right) = \phi - \lambda r \quad ; \qquad \text{LM function .} \tag{2.6}$$

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(3) W. Gibson [17].

By using a simplified version of I. Fisher's interest rate equation, the relationship between the real and nominal interest rates may be expressed as:[4]

$$r = \rho + \pi^e \quad . \tag{2.7}$$

The nominal rate of interest is equal to the sum of the real interest rate and the expected rate of inflation. The expected rate of inflation is used here, and not the actual inflation rate, as people do not have perfect foresight and the nominal interest rate is determined in the money market. Since people do not have perfect foresight, there is a lag between the expected inflation rate and the actual inflation rate causing the nominal rate of interest to have a lagged adjustment to the actual inflation rate.

The determination of inflationary expectations mentioned above are specified in two parts through a linear adjustment process. The first part is Cagan's adaptive expectations equation and the second part is an adjustment towards the government controlled inflation rate, $\pi_g$.

$$\dot{\pi}^e = \beta(\pi-\pi^e) + \delta(\pi_g-\pi) \ ; \qquad \beta > 0, \quad \delta \geq 0 \ . \tag{2.8}$$

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(4) Ibid.

The first term states that when the actual rate of inflation is above the expected inflation rate people will revise their expectations upward. The second term states that when the actual inflation rate is above the controlled inflation rate people will revise their expectations downward as they expect the actual rate to approach the controlled rate. The extent of the effect the control program has on expectations depends on the size of the parameter $\delta$. When $\delta$ is small, expectations are not strongly affected by the government inflation control program. This occurs when people do not believe the controls have much effect on the actual inflation rate.

In this model the government has no direct control over inflation since there is no mechanism forcing the actual inflation rate to comply with the controlled inflation rate. The control program works in the following way. The government states a rate of inflation it would like to prevail. People have expectations of whether the economy will attain this rate of inflation or not. The government controlled inflation rate enters into the model only through these expectations.

When $\beta$ is large (small), people quickly (slowly) adjust the portion of their expectations based on the difference between the actual inflation rate and their expected inflation rate. A small value of $\beta$ provides some stability in inflationary expectations because people wait to see if the change in the actual inflation rate is permanent or not. This means expectations do not react very much to a small change in inflation.

Expectations affect output through Taylor's natural rate of output equation:[5]

$$\ell n\ y = \ell n\ \nu + \gamma(\pi - \pi^e)\ ; \qquad \gamma > 0\ . \tag{2.9}$$

This process, previously explained, relates to the Phillips curve. The parameter $\gamma$ is the speed of adjustment coefficient of output to the difference between the actual and expected rate of inflation. The larger the value of $\gamma$, the faster the output adjusts to the difference between the two inflation rates.

A summary of Model 1 is presented below:

$$\ell n\ y = \alpha - \mu\rho\ ; \qquad \mu > 0 \tag{2.5}$$

$$\ell n\ \left(\frac{M}{Py}\right) = \phi - \lambda r\ ; \qquad \lambda > 0 \tag{2.6}$$

$$r = \rho + \pi^e \tag{2.7}$$

$$\dot{\pi}^e = \beta(\pi - \pi^e) + \delta(\pi_g - \pi)\ ;\ \beta > 0,\ \delta \geq 0 \tag{2.8}$$

$$\ell n\ y = \ell n\ \nu + \gamma(\pi - \pi^e)\ ; \quad \gamma > 0\ . \tag{2.9}$$

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[5] D. Taylor [38].

## B. THE SHORT-RUN

The short-run is the time period during which an economy is adjusting towards a steady-state equilibrium due to the implementation of an inflation control program. For simplicity, both the product and money markets are assumed to be in equilibrium throughout the adjustment process. This does not change the new equilibrium position attained, but the short-run behavior may be different. Since expectations are not directly measurable, they are eliminated from the system by the interest rate equation, Eq (2.7).

Differentiating both the IS and LM functions, Eq (2.5) and Eq (2.6), with respect to time yields:

$$\frac{\dot{y}}{y} = -\mu\dot{\rho} \tag{2.10}$$

$$\frac{\dot{M}}{M} - \pi - \frac{\dot{y}}{y} = -\lambda\dot{r} . \tag{2.11}$$

Eq (2.10), the dynamic product market equilibrium equation, says the actual growth rate of output, $\frac{\dot{y}}{y}$, must equal the negative value of the time derivative of the real interest rate times its sensitivity coefficient.

The rate of change in the money supply is assumed to be exogenously controlled by the monetary authorities. To maintain equilibrium in the money market, the sum of the actual growth rate of output and the inflation rate minus the change in the nominal interest rate at each moment of time multiplied by its sensitivity coefficient must equal the rate of change in the money supply. This is shown by Eq (2.11).

Eliminating the growth rate of output by substituting Eq (2.10) into Eq (2.11) results in the dynamic IS-LM equilibrium equation:

$$\frac{\dot{M}}{M} = \pi - \mu\dot{\rho} - \lambda\dot{r} \quad . \tag{2.12}$$

Replacing $\pi^e$ by Eq (2.7) in the expectations formation equation, Eq (2.8), will result in the following equation:

$$\dot{r} - \dot{\rho} = (\beta-\delta)\pi - \beta r + \beta\rho + \delta\pi_g \quad . \tag{2.13}$$

Since the natural level of output is assumed to be fixed, let $\rho_\nu$ be the corresponding natural real interest rate such that the product market is in equilibrium. Eq (2.5) becomes $\ell n\ \nu = \alpha - \mu\rho_\nu$. By replacing $\ell n\ \nu$ with this expression in Eq (2.9), and by eliminating $\pi^e$, we can solve for $\pi$ :

$$\pi = \frac{\mu}{\gamma}\rho_\nu + r - (1 + \frac{\mu}{\gamma})\rho \quad . \tag{2.14}$$

Solving for $\dot{r}$ and $\dot{\rho}$ in terms of all other variables, by Eq (2.12) and Eq (2.13), and using Eq (2.14) to eliminate $\pi$, results in two equations in state variable form:[6]

$$\dot{r} = \frac{1}{\lambda+\mu}[(1-\mu\delta)r - \{1-\mu\delta + \frac{\mu}{\gamma}(1+\mu(\beta-\delta))\}\rho$$
$$+ \frac{\mu}{\gamma}\{1+\mu(\beta-\delta)\}\rho_\nu - \frac{\dot{M}}{M} + \mu\delta\pi_g] \tag{2.15}$$

$$\dot{\rho} = \frac{1}{\lambda+\mu}[(1+\lambda\delta)r - \{1+\lambda\delta + \frac{\mu}{\gamma}(1-\lambda(\beta-\delta))\}\rho$$
$$+ \frac{\mu}{\gamma}(1-\lambda(\beta-\delta))\rho_\nu - \frac{\dot{M}}{M} - \lambda\delta\pi_g] \quad . \tag{2.16}$$

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[6] See Appendix I-A for the derivation of these equations.

## 1. Stability Conditions

The short-run model has now been reduced to two linear differential equations, Eq (2.15) and Eq (2.16). Applying the Routh-Hurwitz theorem to these equations yields the following necessary and sufficient condition for stability:[7]

$$\lambda\beta < 1 + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda) \quad .$$

It is easily seen that this condition is less restrictive than Cagan's condition. The two positive extra terms on the right-hand side of this condition represents the stabilizing effect of the inflation controls on the model. The first term represents the effect of the control program on the demand for money. The second term represents the effect of the program on the interaction of the IS and LM functions. By increasing the demand for real balances and lowering consumption and investment, the program helps stabilize the economy i.e., the program prevents the destabilizing demand pressures on the money and product markets. As the effect of government controls on expectations becomes smaller, these two terms approach zero, and the stability condition becomes identical to Cagan's, $\lambda\beta < 1$. The stability condition shows the importance of the size of the expectations adjustment coefficients. Larger values of $\delta$ tend to stabilize the economy, while larger values of $\beta$ have a destabilizing effect on the economy.

If the stability condition is violated and the rate of change in the money supply is increased, a new equilibrium position is not attained. People flee from money and increase aggregate demand via

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(7) See Appendix I-B.

the consumption of durable goods. This forces the actual inflation rate, and both interest rates to continuously rise, and the level of output to fall.

The analysis of the effect of a government inflation control program depends upon the stability condition. It is assumed throughout the following discussion that this stability condition is satisfied unless it is stated otherwise.

## 2. Time Paths and Phase Diagrams

A government price control program is applied to an economy at an initial equilibrium position. This initial equilibrium position may or may not have been stable in the pre-control period. During the pre-control period it can be assumed that $\pi = \pi_g$, so the expectations formation equation, Eq (2.8), simplifies to an adaptive expectations expression. D. Taylor's model [38] summarized in the previous chapter is based on the simplified version only.

An inflation control program is applied to the system in an attempt to lower the actual rate of inflation below the rate of change in the money supply. Without a control, the actual rate of inflation equals the growth rate of the money supply. Therefore, this study is mainly devoted to the case where $\pi_g < \frac{\dot{M}}{M}$. The other two cases, $\pi_g = \frac{\dot{M}}{M}$ and $\pi_g > \frac{\dot{M}}{M}$, are briefly discussed.

The short-run adjustment path is the dynamic time path the system follows towards the steady-state or long-run equilibrium position. The system takes an asymptotic path towards an equlibrium if the roots of the characteristic equation are negative and real, and oscillates toward the equilibrium point if the roots are

complex and the real part is negative.[8]

To facilitate the study of the effect of a change in $\pi_g$ on $r$ and $\rho$, phase diagrams in $r$-$\rho$ space are employed. The slopes of both the $\dot{r} = 0$ curve and the $\dot{\rho} = 0$ curve are indeterminate as shown below:

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} = 1 + \frac{\frac{\mu}{\gamma}(1 + \mu(\beta-\delta))}{1-\mu\delta} \gtrless 0 \quad ,$$

$$\left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} = 1 + \frac{\frac{\mu}{\gamma}(1 - \lambda(\beta-\delta))}{1+\lambda\delta} \gtrless 0 \quad .$$

The direction of the shift for the $\dot{r} = 0$ curve is indeterminate while the $\dot{\rho} = 0$ curve shifts down, in $r$-$\rho$ space, when $\pi_g < \frac{\dot{M}}{M}$. This is seen by setting $\dot{r} = 0$ in Eq (2.15) and differentiating $r$ with respect to $\pi_g$.

$$\left.\frac{\partial r}{\partial \pi_g}\right|_{\dot{r}=0} = \frac{-\mu\delta}{1-\mu\delta} \gtrless 0 \quad .$$

Therefore, the direction of the shift of the $\dot{r} = 0$ curve depends on the sign of $1-\mu\delta$. Setting $\dot{\rho} = 0$ in Eq (2.16) and differentiating $r$ with respect to $\pi_g$ results in the $\dot{\rho} = 0$ curve shifting up (down) in $r$-$\rho$ space for a positive (negative) change in $\pi_g$.

$$\left.\frac{\partial r}{\partial \pi_g}\right|_{\dot{\rho}=0} = \frac{\lambda\delta}{1+\lambda\delta} > 0 \quad .$$

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[8] See Appendix I-B.

The relative magnitude of the shifts of the two curves depends upon the sign of $1-\mu\delta$. Therefore two cases exist.(9)

a. Case: $1-\mu\delta < 0$.

This is the case when both $\dot{r} = 0$ and $\dot{\rho} = 0$ curves shift down for a downward change in $\pi_g$. Suppose the $\dot{r} = 0$ curve shifts less than the $\dot{\rho} = 0$ curve. Then,

$$\left.\frac{\partial r}{\partial \pi_g}\right|_{\dot{r}=0} < \left.\frac{\partial r}{\partial \pi_g}\right|_{\dot{\rho}=0} \quad .$$

i.e., $$-\frac{\mu\delta}{1-\mu\delta} < \frac{\lambda\delta}{1+\lambda\delta} \quad .$$

This simplifies to:

$$\mu + \lambda < 0 \quad .$$

Given the restrictions on the parameters, which are both positive, $\mu+\lambda > 0$ must hold. Therefore, the $\dot{r} = 0$ curve must shift more than the $\dot{\rho} = 0$ curve when $1-\mu\delta < 0$.

b. Case: $1-\mu\delta > 0$.

This is the case where $\dot{r} = 0$ curve shifts up and $\dot{\rho} = 0$ curve shifts down, for a downward change in $\pi_g$. Suppose the $\dot{r} = 0$ curve shifts more than the $\dot{\rho} = 0$ curve. Then,

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(9) The special case when $1-\mu\delta = 0$ is briefly examined in Appendix I-C.

$$\left.\frac{-\partial r}{\partial \pi_g}\right|_{\dot{r}=0} > \left.\frac{\partial r}{\partial \pi_g}\right|_{\dot{\rho}=0}$$

i.e.,
$$\frac{\mu\delta}{1-\mu\delta} > \frac{\lambda\delta}{1+\lambda\delta} .$$

This simplifies to:

$$\mu - \lambda + 2\mu\lambda\delta > 0 .$$

Given the restrictions on the parameters we have $\mu-\lambda+2\mu\lambda\delta \gtrless 0$. Therefore, the $\dot{r} = 0$ curve may or may not shift more than the $\dot{\rho} = 0$ curve when $1-\mu\delta > 0$.

Given the indeterminate slopes for $\dot{\rho} = 0$ and $\dot{r} = 0$, and given that $1-\mu\delta \gtrless 0$, five possible cases exist, as shown in Figures 2.1 - 2.5. The arrows represent the directional movement about the new equilibrium point.(10) The solid lines are the initial curves while the dashed lines are the new curves after the implementation of an inflation control program, with $\pi_g < \pi$.

The cases may be identified in the following manner. Subtracting the slopes of the two curves yields:

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} - \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} = \frac{\mu\beta(\mu+\lambda)}{\gamma(1+\lambda\delta)(1-\mu\delta)} .$$

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(10) The direction of the arrows correspond to the signs of the coefficients in Eqs (2.15) and (2.16). When the sign of the coefficient associated with the variable $r$ in Eq (2.15) is positive (negative), the vertical movement is away from (towards) the $\dot{r} = 0$ curve. When the sign of the coefficient associated with the variable $\rho$ in Eq (2.16) is positive (negative), the horizontal arrows point away (towards) the $\dot{\rho} = 0$ curve.

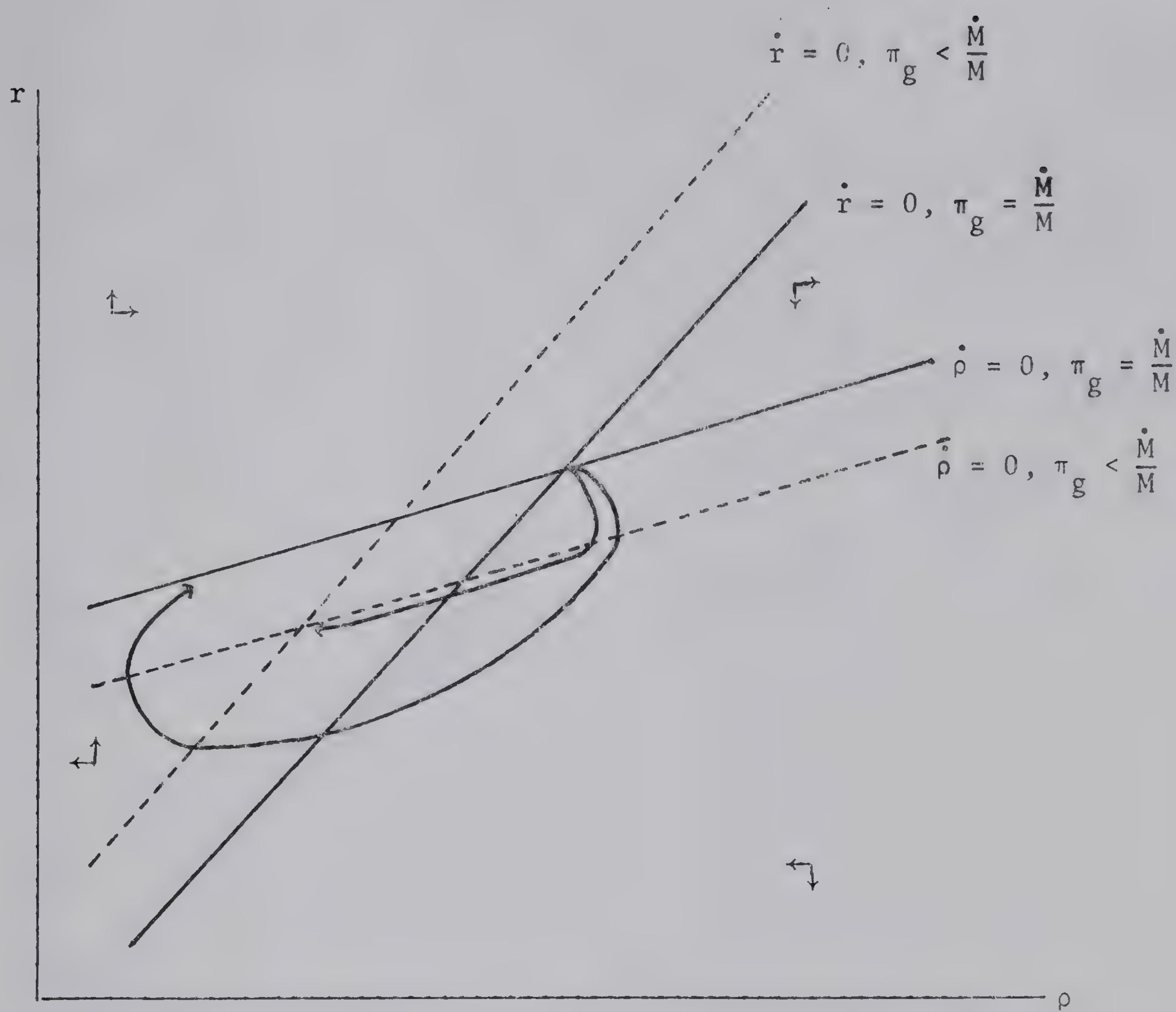


Figure 2.1

Phase Diagram for $\pi_g < \frac{\dot{M}}{M}$

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=o} > \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=o} > 0$$

$$1-\mu\delta > 0$$

$$\lambda\beta < 1 + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda)\ ^{\dagger}$$

†Note: There is a possibility of the trace having a positive value, i.e., $\lambda\beta > 1 + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda)$. The unstable movement away from the equilibrium position is not shown in the diagram.

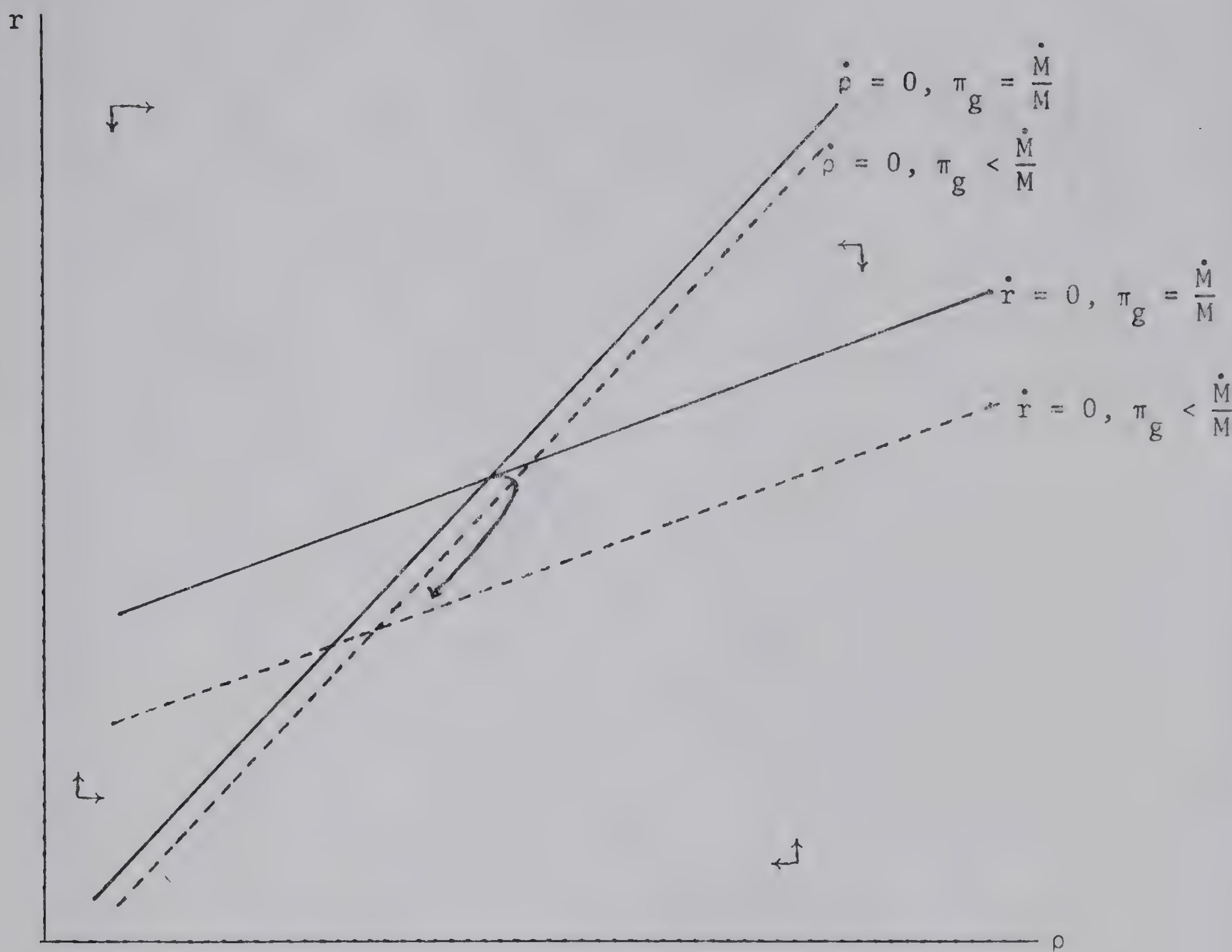


Figure 2.2

Phase Diagram for $\pi_g < \frac{\dot{M}}{M}$

$$\left.\frac{dr}{d\rho}\right|_{\dot{\rho}=o} > \left.\frac{dr}{d\rho}\right|_{\dot{r}=o} > 0$$

$$1 - \mu\delta > 0$$

$$\lambda\beta < 1 + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda)$$

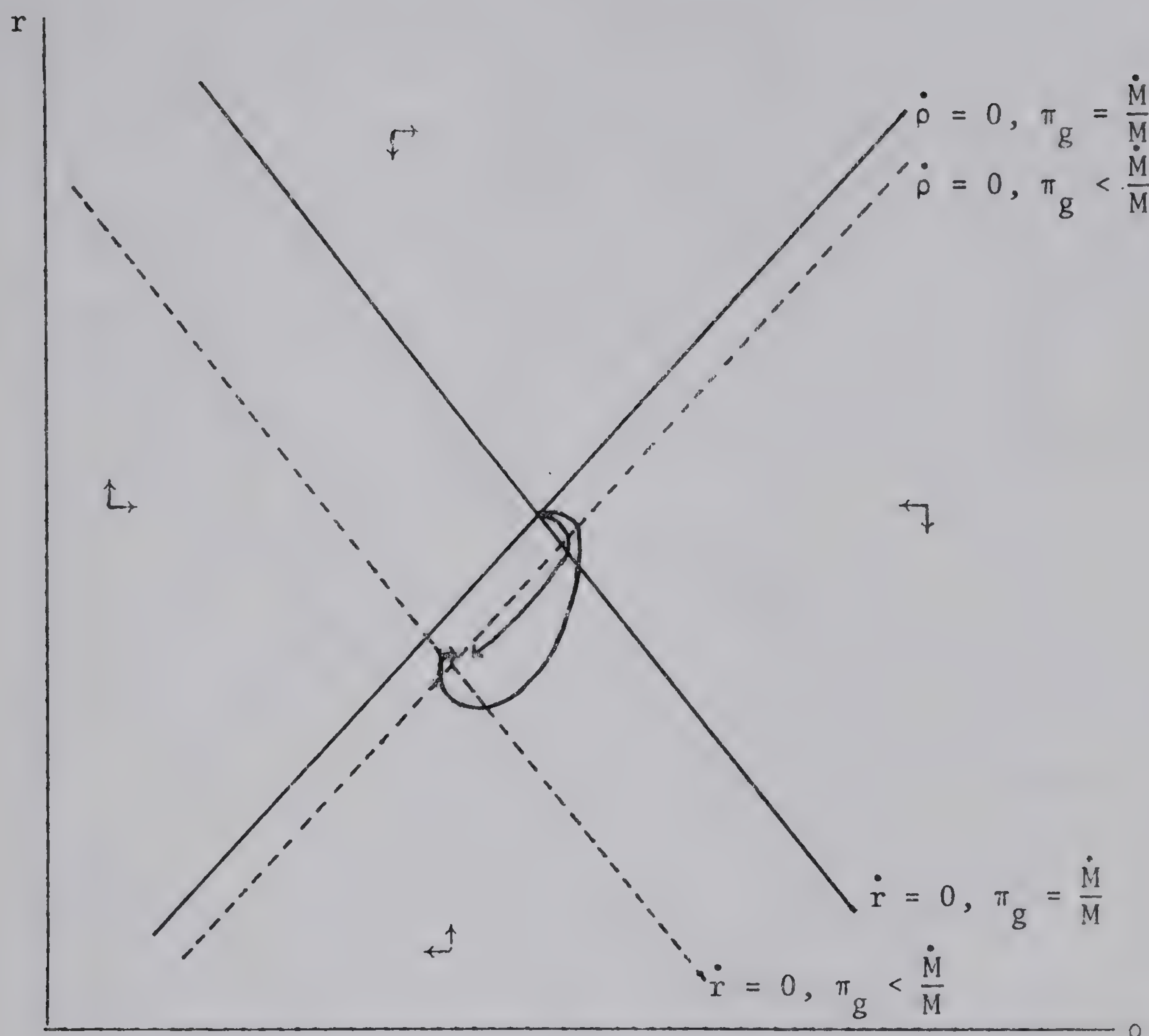


Figure 2.3

Phase Diagram for $\pi_g < \frac{\dot{M}}{M}$

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=o} < 0, \quad \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=o} > 0$$

$$1-\mu\delta < 0$$

$$\lambda\beta < 1 + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda)$$

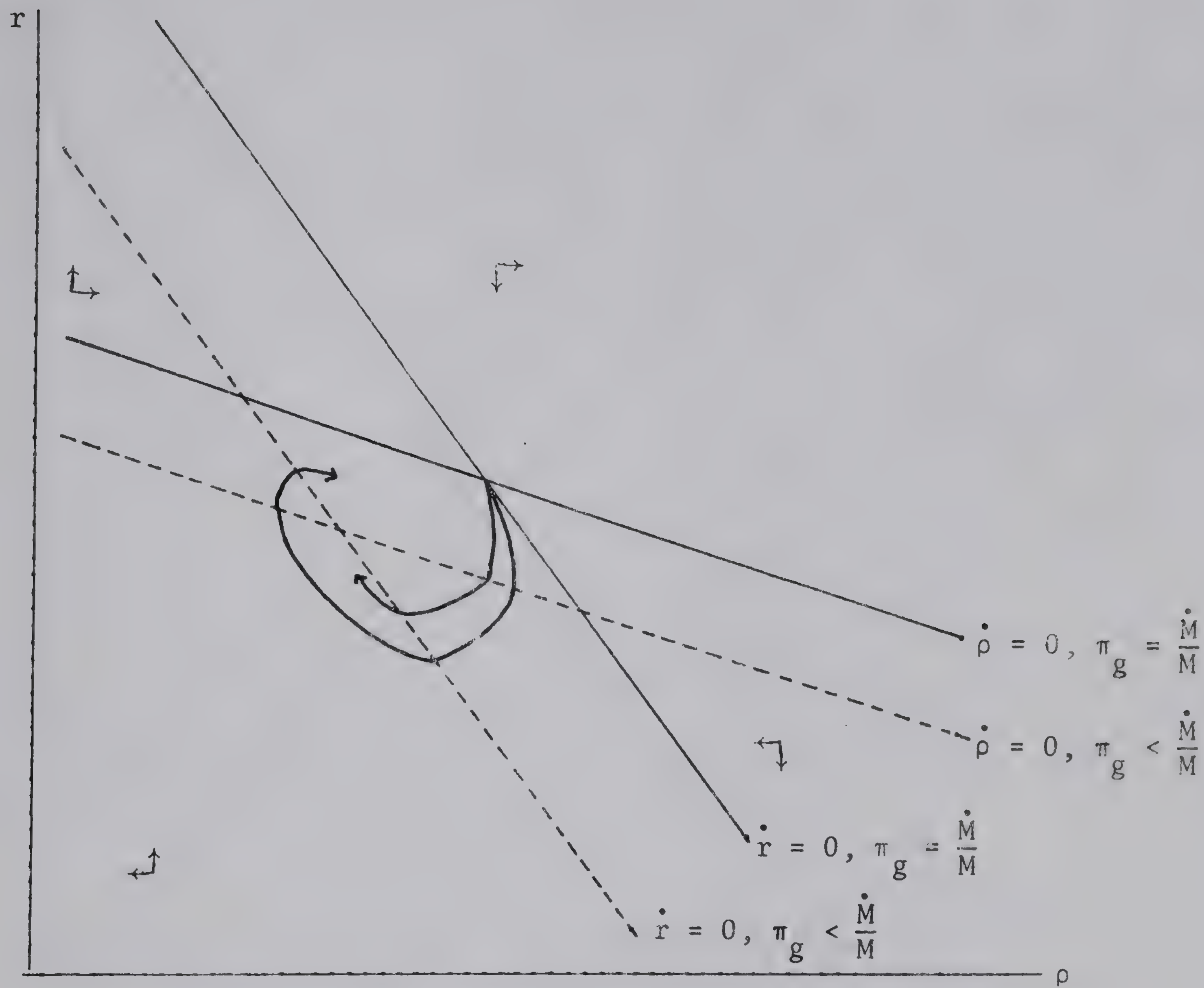


Figure 2.4

Phase Diagram for $\pi_g < \frac{\dot M}{M}$

$$\left.\frac{dr}{d\rho}\right|_{\dot r=o} < \left.\frac{dr}{d\rho}\right|_{\dot\rho=o} < 0$$

$$1-\mu\delta \quad < \quad 0$$

$$\lambda\beta \quad < 1 + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda)^{\dagger}$$

†Note: There is a possibility of the trace having a positive value, i.e., $\lambda\beta > 1 + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda)$. The unstable movement away from the equilibrium position is not shown in the diagram.

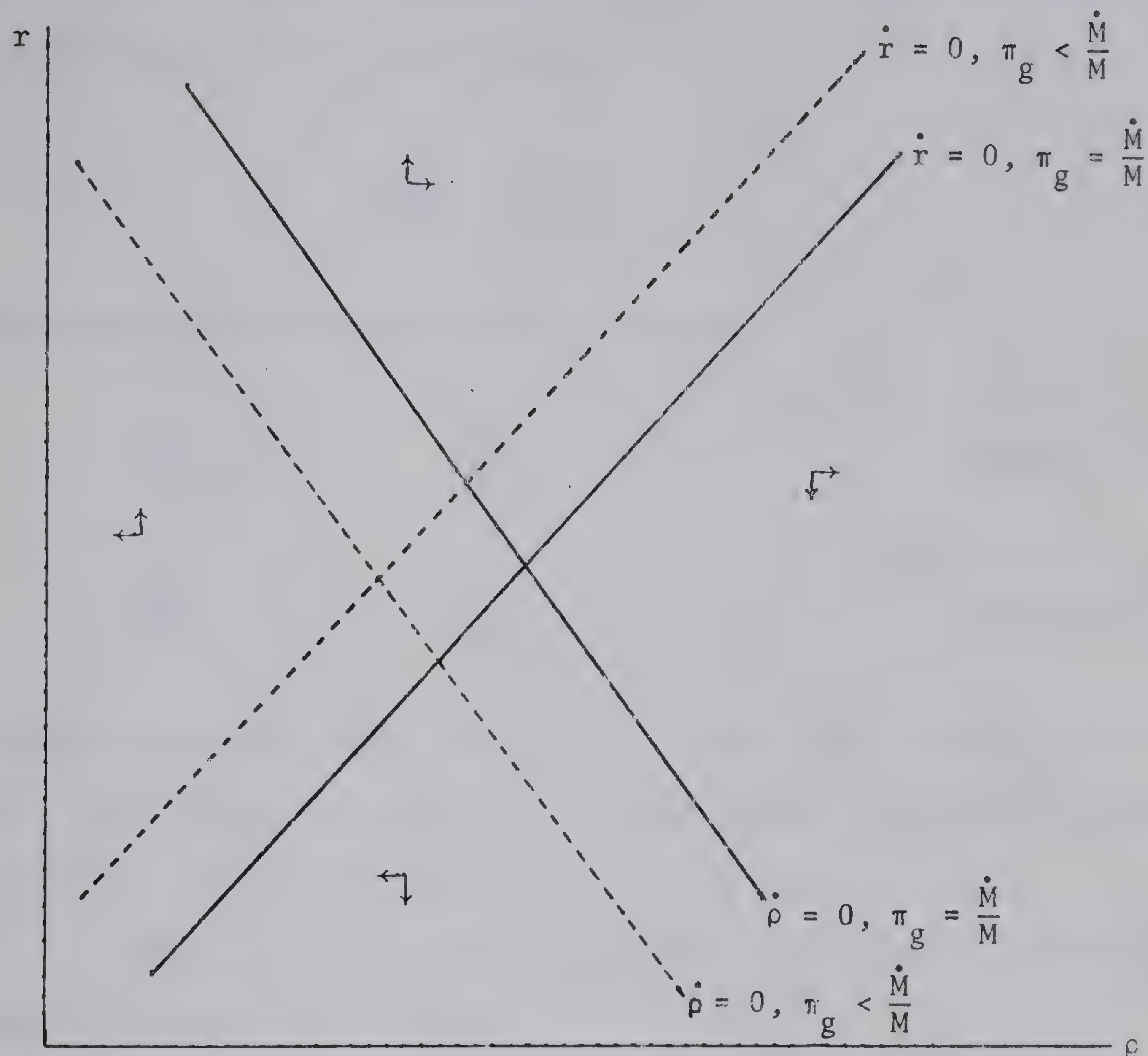


Figure 2.5

Phase Diagram for $\pi_g < \frac{\dot{M}}{M}$

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=o} > 0, \quad \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=o} < 0$$

$$1-\mu\delta > 0$$

$$\lambda\beta > 1 + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda)$$

When $1-\mu\delta$ is positive, the slope of the $\dot{r} = 0$ curve is greater than the slope of the $\dot{\rho} = 0$ curve:

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} > \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} .$$

Two possible cases result from this condition:

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} > 0 , \qquad \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} > 0 , \qquad \text{(Figure 2.1)}$$

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} > 0 , \qquad \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} < 0 . \qquad \text{(Figure 2.5)}$$

Upon inspecting the slope of the $\dot{r} = 0$ curve, one can easily see that when $1-\mu\delta$ is positive, the $\dot{r} = 0$ curve cannot be downward sloping. Therefore, these are the only possible cases when $1-\mu\delta > 0$.

When $1-\mu\delta < 0$, the slope of the $\dot{r} = 0$ curve is less than the slope of the $\dot{\rho} = 0$ curve:

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} < \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} .$$

This results in the three following cases:

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} > 0 , \qquad \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} > 0 , \qquad \text{(Figure 2.2)}$$

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} < 0 , \qquad \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} > 0 , \qquad \text{(Figure 2.3)}$$

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} < 0 , \qquad \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} < 0 . \qquad \text{(Figure 2.4)}$$

Two of the above cases, Figures 2.2 and 2.3 are always stable, while two may be stable, Figures 2.1 and 2.4. Three of the cases, Figures 2.1, 2.3 and 2.4, allow oscillatory movements towards the new equilibrium. All four stable cases allow an asymptotic approach towards the equilibrium position. The possible unstable cases, in Figures 2.1, 2.4 and 2.5, are not discussed here.

In the four stable cases, aggregate demand falls initially, causing the nominal interest rate to fall. The real interest rate increases causing output to fall, then falls, causing output to rise. The case which prevails depends upon the sizes of the parameter values. Similarly, the sizes of the shifts of both curves depend on the parameters. In all cases, the movement is in a clockwise direction. Graphs of the movement of typical examples of the variables $r$ and $\rho$, plotted over time, are shown in Figures 2.6 - 2.7. Similar graphs for output and the two inflation rates are also shown in these Figures.

The initial movement of the interest rates has already been discussed. In the initial stages of the control program, the level of output falls below its natural level in all four stable cases. Both the actual and expected inflation rates fall initially also. The movement of the variables is in the following order: after the real interest rate begins to fall, the nominal interest rate leads the real interest rate, followed by output, then the actual inflation rate, with expectations lagging behind the actual inflation rate. In the cyclical cases, this means peaks and troughs of the nominal interest rate occur before those of the real interest rate. In turn, the level of output reachs its maximum or minimum points at the same time as the real

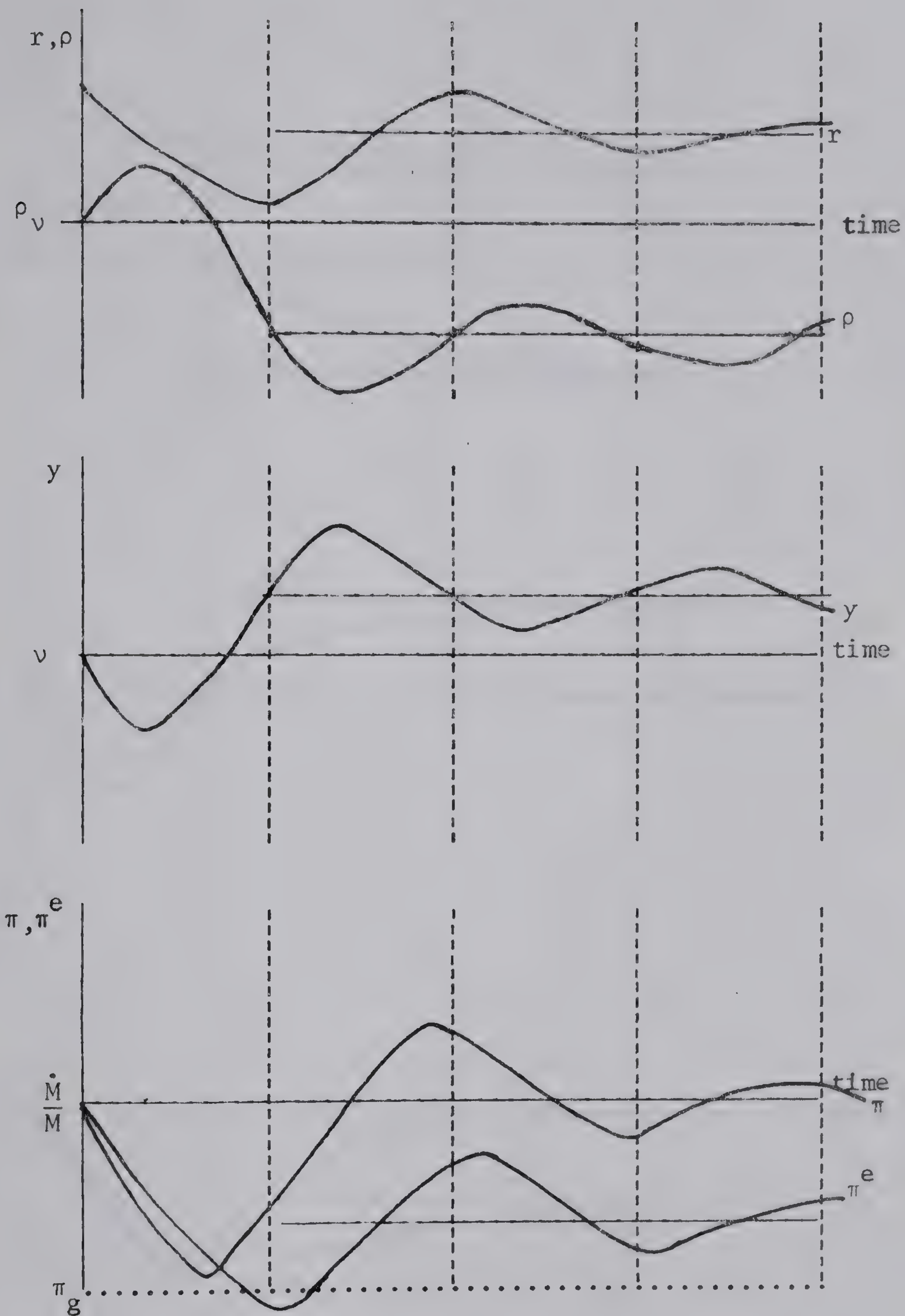


Figure 2.6

Time Paths for $\pi_g < \frac{\dot{M}}{M}$ : Cyclical Case†

†Note: These time paths, corresponding to Figures 2.2 - 2.4, are for a typical case only. Other oscillatory paths may occur.

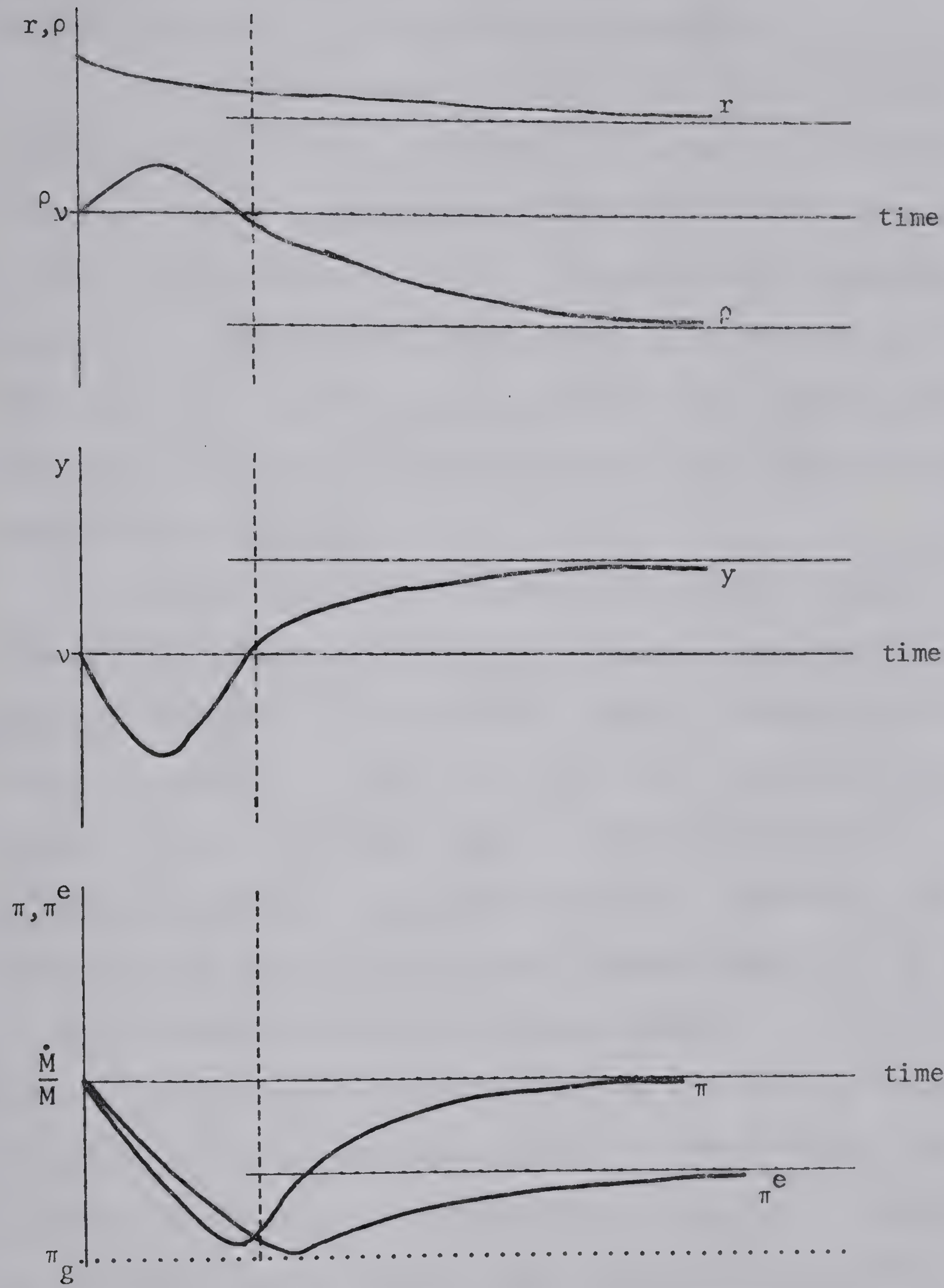


Figure 2.7

Time Paths for $\pi_g < \frac{\dot{M}}{M}$: Asymptotic Case†

†Note: These time paths, corresponding to Figures 2.1 - 2.4, are for a typical case only.

interest rate. The actual and expected inflation rate follow behind respectively. This is shown in Figure 2.6.

Initially the actual inflation rate falls below the expected inflation rate, and expectations adjust to the actual inflation rate in a lagged manner. While this occurs, output remains below its natural level. The point where expectations equals the actual inflation rate, output is at its natural level. After this point, in the two cases shown, output remains above its natural level and the actual inflation rate remains above the expected inflation rate.

Since output and the nominal interest rate decline, the effect of the inflation control program on real money balances is indeterminate initially. This is because the falling level of output puts downward pressure on the demand for real balances while the falling nominal interest rate tends to increase the demand for real money balances. Therefore, the effect on the demand for real money balances depends on the size of the decrease in the level of output relative to a nominal interest rate times the parameter $\lambda$. By the time the level of output has risen to its natural level, the level of real money balances is above its pre-control level in any case. Eventually, all variables approach their long-run equilibrium positions. The long-run equilibrium positions are discussed at length in Section C.

3. The Effect of Inflation Controls When $\pi_g \geq \frac{\dot{M}}{M}$

An inflation control program is established in an economy at a long-run equilibrium position. When the controlled inflation rate is set equal to the actual inflation rate, the program is ineffective, because the control does not alter expectations. This can be seen in Eq (2.8) when $\pi_g = \pi$. Therefore, the economy remains at the same equilibrium position.

When the controlled inflation rate is set above the growth rate of the money supply, two events may occur. People may ignore the control, $\delta = 0$; therefore the control remains as ineffective as when $\pi_g = \pi$. Another possibility exists in this case. People may place some value on this controlled rate and expect inflation to rise. Entrepreneurs may use the higher controlled rate as an excuse to increase prices and labourers may use it for higher wage demands which are met by entrepreneurs. The higher wages are passed on to consumers by higher prices. If this occurs, Figures 2.1 - 2.5 are reversed. The dashed lines are the initial curves while the solid lines are the new curves. Initially the nominal interest rate, the level of output and both the actual and expected inflation rates all rise. On the other hand, the real rate of interest falls initially. The final equilibrium positions of the variables are discussed in the next section. Reid [33] examines this case where the application of an inflation control program causes the actual rate of inflation to rise to equal the controlled inflation rate.

## C. THE LONG-RUN EQUILIBRIUM POSITION

The long-run equilibrium position for the actual inflation rate is obtained by setting the time derivatives of $r$ and $\rho$ equal to zero in Eq (2.12). Equating $\dot{\pi}^e$ to zero, one can solve for the equilibrium value of $\pi^e$ in Eq (2.8). By solving the simultaneous system of equations, Eq (2.15) and Eq (2.16) with $\dot{r} = \dot{\rho} = 0$, the equilibrium values of $r$ and $\rho$ are obtained. Replacing the equilibrium values of $\pi$ and $\pi^e$ in Eq (2.9) yields the long-run value of $\ell n\ y$. The long-run value of real balances is acquired by substituting the equilibrium values of $\ell n\ y$, $r$, and $\pi$ into Eq (2.6). The resulting equilibrium values are the following:

$$\pi = \frac{\dot{M}}{M}$$

$$\pi^e = \frac{\dot{M}}{M} - \frac{\delta}{\beta}\left(\frac{\dot{M}}{M} - \pi_g\right)$$

$$\rho = \rho_\nu - \frac{\gamma\delta}{\beta\mu}\left(\frac{\dot{M}}{M} - \pi_g\right)$$

$$r = \rho_\nu + \frac{\dot{M}}{M} - \frac{\delta(\gamma+\mu)}{\beta\mu}\left(\frac{\dot{M}}{M} - \pi_g\right)$$

$$\ell n\ y = \ell n\ \nu + \frac{\gamma\delta}{\beta}\left(\frac{\dot{M}}{M} - \pi_g\right)$$

$$\ell n\ \left(\frac{M}{P}\right) = \ell n\ \left(\frac{M}{P}\right)_1 + \left[\frac{\delta(\gamma+\lambda)}{\beta} + \frac{\lambda\delta\gamma}{\beta\mu}\right]\left(\frac{\dot{M}}{M} - \pi_g\right)$$

where

$$\ell n\ \left(\frac{M}{P}\right)_1 = \phi - \lambda\left(\rho_\nu + \frac{\dot{M}}{M}\right) + \ell n\ \nu\ .$$

The first observation is that the rate of inflation equals the rate of change in the money supply. This means that the control program has no effect on inflation in the long-run. The second observation is that all other variables are affected by the controls resulting in equilibrium positions that may differ from those in the uncontrolled state. The solutions depend upon the relative sizes of $\frac{\dot{M}}{M}$ and $\pi_g$. Three cases exist.

1. Case: $\pi_g < \frac{\dot{M}}{M}$ .

In this case the government sets their controlled inflation rate below the rate of growth in the money supply. The equilibrium position corresponds to the new equilibrium positions in Figures 2.1 - 2.4. The exact position of the equilibrium depends on the extent people base their expectations on $\pi_g$, the sizes of the parameters $\delta$ and $\beta$, as well as the difference between $\frac{\dot{M}}{M}$ and $\pi_g$. As $\delta$ increases or $\beta$ decreases the controlled equilibrium position is further away from its intial equilibrium position.

We have assumed portfolios are in equilibrium as the nominal interest rate equals the sum of the real interest rate and expected rate of inflation. In this case both interest rates and the expected inflation rate are in equilibrium below their uncontrolled equilibrium values. The real level of output is above the natural level, and real cash-balances are above the uncontrolled equilibrium level. (See Figures 2.6 - 2.7.)

2. Case: $\pi_g = \frac{\dot{M}}{M}$ .

When the government controlled rate is set equal to the rate of change in the money supply, the long-run equilibrium is the same equilibrium as that attained without a control program. The long-run results are also those that are attained if people do not believe in controls, i.e., $\delta = 0$. In this case the equilibrium values simplify to:

$$\pi = \frac{\dot{M}}{M} = \pi^e$$

$$\rho = \rho_\nu$$

$$r = \rho_\nu + \frac{\dot{M}}{M}$$

$$\ln y = \ln \nu$$

$$\ln \left(\frac{M}{P}\right) = \ln \left(\frac{M}{P}\right)_1 = \phi - \lambda r + \ln \nu .$$

The real interest rate and the level of output are both at their natural levels, and expectations are realized (expectations equal to actual inflation rate). The nominal interest rate equals the natural real interest rate plus the rate of inflation. The level of real cash-balances is at the same long-run level as it is in the uncontrolled economy.

3. Case: $\pi_g > \frac{\dot{M}}{M}$ .

Two different possible equilibrium positions may exist under this control program. If the program has no influence on expectations,

$\delta = 0$, the equilibrium position is the same one as previously discussed.

If people use the control rate as a bargaining position, the resulting equilibrium is opposite to that when $\pi_g < \frac{\dot{M}}{M}$. The size of the parameters $\delta$ and $\beta$ have the same effect on the position of the equilibrium as stated earlier. People expect a higher inflation rate so $\pi^e > \pi$. Both interest rates are above their pre-control rates while the level of output and real cash-balances are below their pre-control levels. The results are easily seen upon inspecting the equilibrium values at the beginning of Section C.

D. THE POST-CONTROL PERIOD

These long-run equilibrium solutions give the eventual outcome of a control program if it remains permanently in place. But often the control program is removed before these equilibrium positions are attained; therefore, an important question is the ability of the economy to regain its equilibrium position after the removal of the inflation control program. For this reason, we must examine this aspect of the program for both a stable and unstable post-control economic system. Emphasis is on the case when $\pi_g < \frac{\dot{M}}{M}$.

Once the inflation control program is removed, $\pi_g = \pi$, and the post-control equilibrium position is the same as the pre-control position. The economy adjusts towards or away from this equilibrium position depending on whether or not the uncontrolled economic system is stable or unstable. The post-control stability condition is Cagan's stability condition, $\lambda\beta < 1$. Examination of the stable case precedes the unstable case.

1. Stable Post-Control Conditions After $\pi_g < \frac{\dot{M}}{M}$ Controls

The $\dot{\rho} = 0$ curve is positively sloped and less than the slope of the $\dot{r} = 0$ curve (in $r$-$\rho$ space) in the uncontrolled model. This means the cases involving positive sloped $\dot{\rho} = 0$ curves during controls, Figures 2.1 - 2.3, are the only cases that will result in a stable post-control equilibrium. The control program increases the stability of the model. The stronger the stabilizing effect the program has on the model, the more the $\dot{\rho} = 0$ curve rotates to become positively sloped when starting from an unstable uncontrolled model. So, a negative sloped $\dot{\rho} = 0$ curve during controls means that the slope of the $\dot{\rho} = 0$ curve in the uncontrolled economy must also be negative. Figures 2.1 - 2.3 allow for the possible approach patterns to the long-run post-control equilibrium shown in Figure 2.8. The arrows represent the directional movement about the equilibrium.(11) The initial starting position is located somewhere in the shaded area below the long-run equilibrium position. The numbers one through four indicate regions of possible starting points. The approach towards the equilibrium may be cyclical or asymptotic for all possible starting positions.

The position of the economy when the control program is removed relative to the new equilibrium position results in various initial movements of the endogenous variables. If the controls are removed when $\rho$ is to the right of $\rho_\nu$ (region one in Figure 2.8),

(11) See Footnote (10) for an explanation of the arrows.

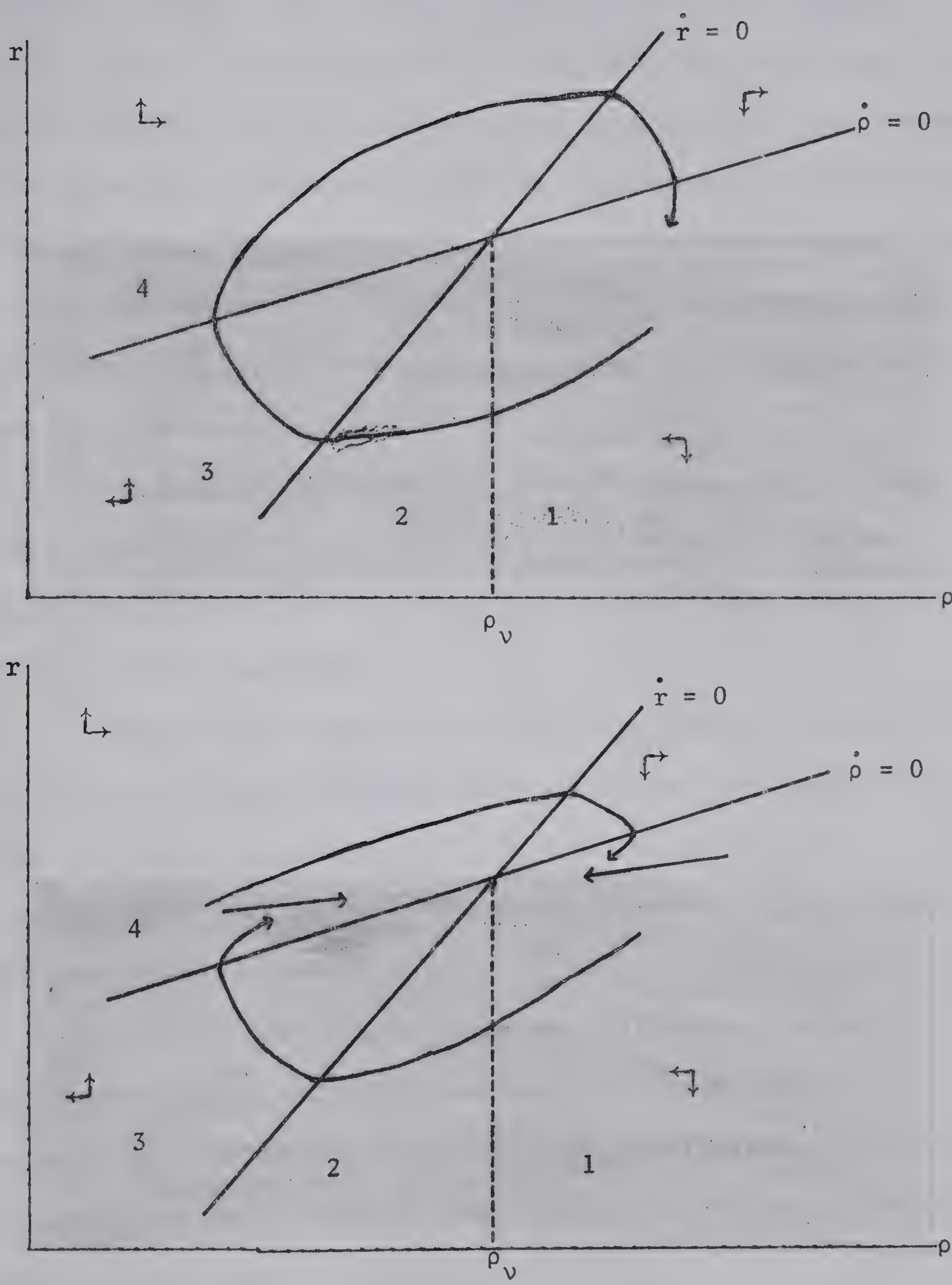


Figure 2.8

Phase Diagrams for a Stable Post-Control Equilibrium $\lambda\beta < 1^{\dagger}$

†Note: The non-dominant separatrix has been omitted from the bottom diagram for clarity.

the nominal interest rate, which is initially below its long-run position, falls. At the same time $\rho$, which is initially above its long-run position, falls. This means that output rises, but is below its long-run level. Both the actual and expected inflation rates are rising, but below their long-run equilibrium positions. Expectations are lower than the actual inflation rate at this time. This corresponds to the removal of the control program early in its stages, or in later stages of cyclical movement.

The same initial movements occur in region two, but here the real rate of interest is starting from a position below its long-run equilibrium, and the inflation rates and output are above their equilibrium positions.

Region three shows the starting point where the nominal interest rate and both inflation rates are below their long-run values, but are rising. The real interest rate is below its equilibrium rate, and falling. The level of output is above its equilibrium level and is rising.

In all three of these cases the initial position of real money balances relative to its long-run equilibrium position is indeterminate. In regions one and two real money balances are rising. The movement in region three is indeterminate as both output and the nominal interest rate are rising.

The final possible starting position is in region four. Here both interest rates are starting from positions below their long-run rates and both are increasing. The level of output, and both inflation rates are above their equilibrium values. The level

of output is falling while the inflation rate is rising. The level of real cash-balances is below its equilibrium level and falling.

2. Unstable Post-Control Condition After $\pi_g < \frac{\dot{M}}{M}$ Controls

When the post-control stability conditions are not met, the $\dot{r} = 0$ curve is still upward sloping but now the $\dot{\rho} = 0$ has a positive slope. All four cases corresponding to Figures 2.1 - 2.4 during the control program may result in an unstable post-control equilibrium situation. As before, once controls are removed the economy is not at its long-run equilibrium position. Figure 2.9 shows the possible movements that may result from the removal of the inflation control program. Again, the shaded area shows the area of possible starting positions after the removal of the control program. The movement may be a cyclical or asymptotic explosive path away from the equilibrium position.

Initially the nominal interest rate is below its equilibrium value while the real interest rate may be above or below its equilibrium value. In region one the nominal interest rate falls while the real interest rate rises. The level of output and both inflation rates fall and the level of real money balances rise. Region two results in both interest rates falling initially while the level of output, and the actual and expected inflation rates rise. The movement of real money balances depends upon the relative sizes of the level of output and the nominal interest rate times its coefficient. Region three results in initial movements where the real interest rate falls while the nominal interest rate, the level of output and both inflation rates rise. The level of real money balances may either rise or fall as in the previous case. If there are cycles, the time path passes into all regions, if not, most regions are never reached.

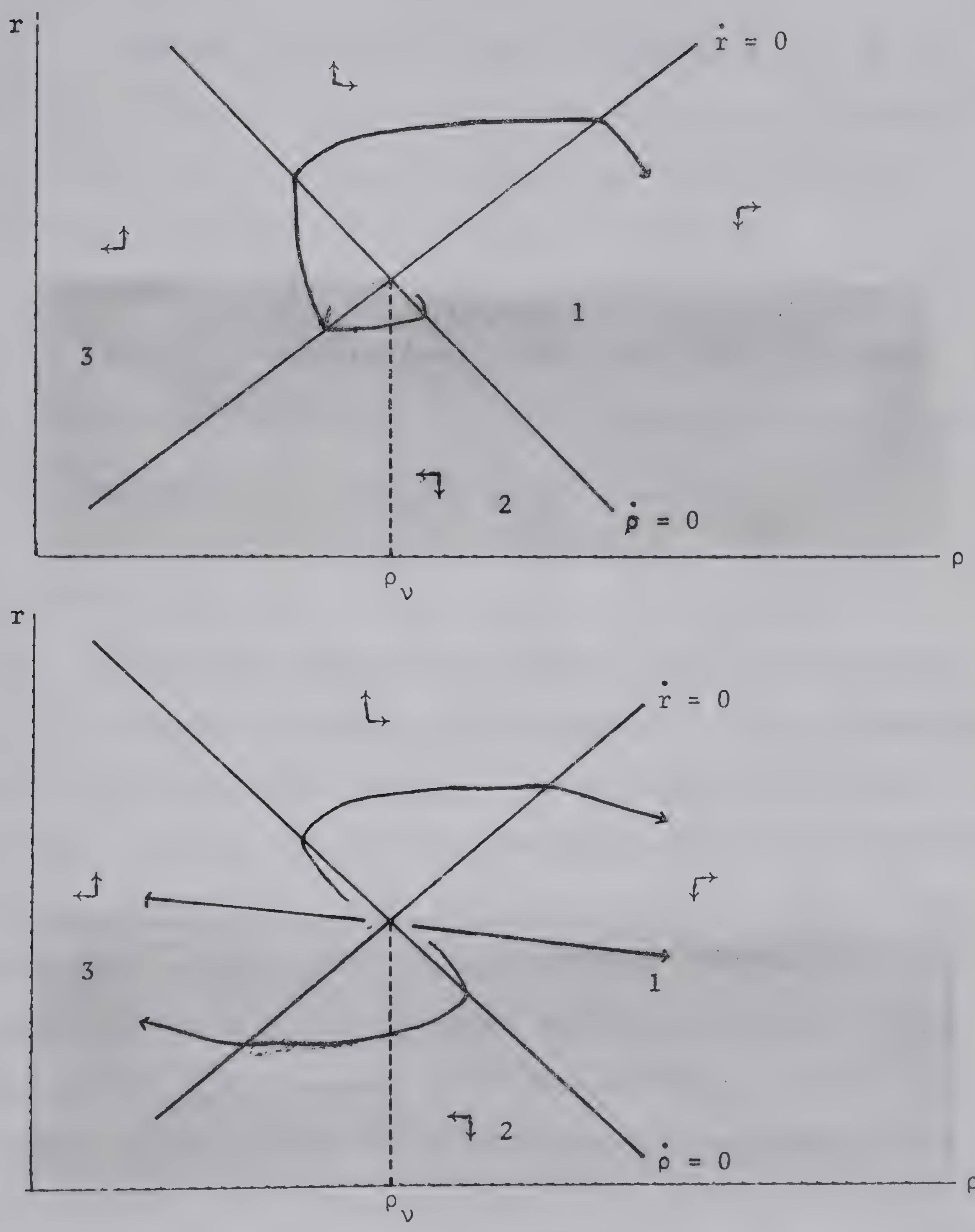


Figure 2.9

Phase Diagram for an Unstable Post-Control Equilibrium $\lambda\beta \geq 1^{\dagger}$

$^{\dagger}$Note: The non-dominant separatrix has been omitted from the bottom diagram for clarity.

3. <u>The Removal of Controls When</u> $\pi_g = \frac{\dot{M}}{M}$ <u>or</u> $\pi_g > \frac{\dot{M}}{M}$

Removing the inflation control program when $\pi_g = \frac{\dot{M}}{M}$ or when $\pi_g > \frac{\dot{M}}{M}$ and $\delta = 0$ does not create any time path movement at all because the control program had no effect on the equilibrium position and therefore left the economy undisturbed.

When $\pi_g > \frac{\dot{M}}{M}$ and $\delta > 0$, the time paths mirror those of $\pi_g < \frac{\dot{M}}{M}$ for both the stable and unstable cases. The pre-control equilibrium positions are above the control period equilibrium instead of below.

E. SUMMARY

The implementation of an inflation control program, where $\pi_g < \frac{\dot{M}}{M}$, causes the economy to follow either a cyclical or asymptotic path to a new equilibrium position. In the long-run equilibrium position, the level of output and real money balances are higher than their pre-control levels while both interest rates and inflationary expectations are below their pre-control rates. The short-run time paths of all the endogenous variables may either fluctuate, or move asymtotically to the long-run equilibrium position. In either case, periods of falling output and inflation exist in the initial stages of the program. Due to the specification on the demand for money function, the long-run actual rate of inflation equals the growth rate of the money supply as it did in the pre-control period.

Once the inflation control program is removed, the long-run equilibrium reverts to its previous position. If the post-control

equilibrium is stable, all the endogenous variables will eventually return to their pre-control equilibrium values. If the post-control equilibrium is unstable, the removal of the control program causes an explosive or cyclical time path away from the equilibrium position.

Even though the program has no long-run effect on inflation, it does raise the level of output temporarily or as long as the control program remains in effect. The program may be beneficial if the benefits from higher output outweigh the costs from the periods of lower output. Since the control program does not alter the long-run rate of inflation, a voluntary control program may not be enough. It may be more effective to implement the program along with a restrictive monetary or fiscal policy. Another possibility is that the voluntary control program alters more than inflationary expectations. This being the case, the program may be effective in lowering the rate of inflation in the long-run. By making an adjustment to another equation, Model 2 attempts to alter the long-run result.

# CHAPTER III

## MODEL 2

Although Model 1 captures the short-run effect of controls on output as described by R. Lipsey [25] and U. Possen [32], it has one major drawback: the actual rate of inflation is unaffected by controls in the long-run while expectations are affected. The reason for this is the inflation control program alters the inflationary expectations equation only. The control program does not alter the equation which determines the long-run inflation rate. In consequence, in the long-run, the level of output is above its natural level, while the actual rate of inflation has returned to its pre-control equilibrium position.

In the most likely case of Model 1 when $\pi_g < \frac{\dot{M}}{M}$, the long-run equilibrium position is one where expectations are lower than the actual inflation rate. Assuming there is no other autonomous shock, Model 1 presumes people are naive enough to believe the controls will bring the inflation rate down even after a considerable length of time of observing the program's ineffectiveness. Presumably, the parameter associated with controls in the expectations formation process would approach zero as people observe the inability of the control program to work. This would cause expectations to rise and output to fall to their pre-control equilibrium positions. Model 2 attempts to rectify the problem of no long-run effect on inflation by allowing the voluntary control program to alter the demand for money.

Since the only equation to be altered from those in Model 1 is the demand for money function, this is the only function to be explained in this chapter. The analysis of the model proceeds as for Model 1. Differential equations are derived for both the real and nominal rates of inflation and the stability condition for the model is examined. Phase diagrams and short-run time paths are analyzed for all possible stable cases, followed by an explanation of the long-run equilibrium positions. Since inflation control programs are temporary in nature, an examination of the post-control period concludes the chapter.

Most inflation control programs set the controlled rate of inflation below the actual rate of inflation. Initially, the actual rate of inflation equals the growth rate of the money supply in the model. Therefore, the discussion centers on the case where the controlled rate of inflation is less than the growth rate of the money supply. The other two cases where the controlled inflation rate is greater than the growth rate of the money supply and the controlled rate equals the growth rate of the money supply are briefly discussed.

## A. CONSTRUCTING THE MODEL

The government implements a voluntary inflation control program. As in Model 1, people expect the inflation rate to move towards the controlled inflation rate. Expectations of the actual inflation rate approaching the controlled rate make people aware that both their income and the nominal interest rate will change in the long-run. It is assumed the demand for money is now a function

of the real and expected levels of output, and the actual and expected nominal interest rates. When the controlled inflation rate is set below the actual rate of inflation, the long-run actual price level would be lower than it would have been if the control program had not been implemented. Therefore, people expect output to rise and the nominal rate to fall in the long-run. Hence, people are willing to hold more money than they would have without the control program.

Both the expected nominal interest rate and the expected level of output are not easily determined. Therefore an approximation for the resulting effect of these two variables is used. We assume that two obserable price levels exist at each point in time. A price ratio of the actual price level over the price level associated with the government control program is used as an approximation of the effect of the expected nominal interest rate and the expected level of output in the following manner. In the uncontrolled model this ratio equals one as the two price levels are identical. For the most part, during the control program the associated price level is less than the actual price level as the controlled inflation rate is less than the actual inflation rate. Therefore, the price ratio is usually greater than one. This causes the demand for the real balances to be higher in the controlled model than the uncontrolled model for a given level of output and a given nominal interest rate. The price ratio creates the desired effect on the demand for money. Therefore, it is used as an approximation of the effect of the expected output level and

expected interest rate have on the demand for money. Specifically, the demand for money function is:

$$\frac{M^d}{P} = y \left(\frac{P}{P_g}\right)^{\varepsilon} e^{\phi - \lambda r} \; ; \quad \lambda > 0, \quad \varepsilon \geq 0 \; . \tag{3.1}$$

The price level associated with the government controlled inflation rate at each moment in time is $P_g$, while $\varepsilon$ is the elasticity of the demand for real money balances with respect to the price ratio $\frac{P}{P_g}$. The larger the value of the parameter $\varepsilon$, the more effect the program has on the demand for money. When the controls are first imposed on the model, $P = P_g$, so the demand for money function is the same as in the uncontrolled model.

As in Model 1, it is assumed that money market equilibrium is maintained at each point in time and the LM curve is transformed into its semi-logarithmic form:

$$\ell n \left(\frac{M}{Py}\right) = \phi - \lambda r + \varepsilon(\ell n\ P - \ell n\ P_g) \; ; \quad \text{LM function.} \tag{3.2}$$

The remaining equations in the model are the same as those used in Model 1. These are summarized below.

$$\ell n\ y = \alpha - \mu\rho \quad ; \quad \mu > 0 \tag{2.5}$$

$$\ln\left(\frac{M}{Py}\right) = \phi - \lambda r + \varepsilon(\ln P - \ln P_g) \; ; \qquad \lambda > 0, \quad \varepsilon \geq 0 \tag{3.2}$$

$$r = \rho + \pi^e \tag{2.7}$$

$$\dot{\pi}^e = \beta(\pi - \pi^e) + \delta(\pi_g - \pi); \qquad \beta > 0, \quad \delta \geq 0 \tag{2.8}$$

$$\ln y = \ln \nu + \gamma(\pi - \pi^e) \; ; \qquad \gamma > 0 \quad . \tag{2.9}$$

The analysis proceeds in the same manner as in Model 1. First, the short-run dynamic time paths are examined when the model is stable. The long-run steady-state equilibrium positions are examined followed by the effects of removing the control program. Emphasis is placed on the case where $\pi_g < \frac{\dot{M}}{M}$. The other two cases, $\pi_g \geq \frac{\dot{M}}{M}$ are briefly discussed.

## B. THE SHORT-RUN

The method of solving the system of equations is the same as that used for Model 1. Upon differentiating the money market equilibrium function, Eq (3.2), with respect to time, we have a term involving the government controlled inflation rate.

$$\frac{\dot{M}}{M} = (1+\varepsilon)\pi - \varepsilon\pi_g - \lambda\dot{r} + \frac{\dot{y}}{y} \quad .$$

Here, both $\frac{\dot{M}}{M}$ and $\pi_g$ are exogenously controlled. It is through these two variables that the government has some control over inflation.

Solving the system of equations, outlined in the summary above, yields the two following equations in state variable form:[12]

$$\dot{r} = \frac{1}{\lambda+\mu}\Big[(1+\varepsilon-\mu\delta)r - \{1+\varepsilon-\mu\delta+\frac{\mu}{\gamma}(1+\varepsilon+\mu(\beta-\delta))\}\rho + \frac{\mu}{\gamma}(1+\varepsilon+\mu(\beta-\delta))\rho_\nu - \frac{\dot{M}}{M} + (\mu\delta-\varepsilon)\pi_g\Big] \qquad (3.3)$$

$$\dot{\rho} = \frac{1}{\lambda+\mu}\Big[(1+\varepsilon+\lambda\delta)r - \{1+\varepsilon+\lambda\delta+\frac{\mu}{\gamma}(1+\varepsilon-\lambda(\beta-\delta))\}\rho + \frac{\mu}{\gamma}(1+\varepsilon-\lambda(\beta-\delta))\rho_\nu - \frac{\dot{M}}{M} - (\lambda\delta+\varepsilon)\pi_g\Big] \quad . \qquad (3.4)$$

1. <u>Stability Conditions</u>

By applying the Routh-Hurwitz theorem to Eqs (3.3) and (3.4) the following necessary and sufficient stability condition is attained:[13]

$$\lambda\beta < 1 + \varepsilon + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda) \quad .$$

Not only is this condition less restrictive than Cagan's condition, but it is also less restrictive than the equivalent condition for Model 1.[14] The three terms on the right-hand side of the condition are due to the stabilizing effect of the inflation controls. The two terms involving $\delta$ are explained in Model 1. The term $\varepsilon$ appears

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[12] The derivation of these equations is presented in Appendix II-A.

[13] See Appendix II-B.

[14] Model 1 is a simplified version of Model 2 where $\varepsilon$ equals zero.

in the stability condition as it appears in the LM functional relationship. The control program helps to prevent people from demanding durable goods instead of holding money, an asset switch which occurs in the unstable, uncontrolled model. The program prevents the inflation rate from continously rising by lowering the demand pressure in the product market, thereby stabilizing the economy. The controlled model is automatically stable if the pre-control stability condition is satisfied. Because this stability condition is less restrictive than that of the uncontrolled model ($\lambda\beta < 1$), applying an inflation control program may have a stabilizing effect on the unstable model.

Throughout the following analysis, it is assumed that the stability condition of our model is satisfied.

2. Time Paths and Phase Diagrams When $\pi_g < \frac{\dot{M}}{M}$

As in Model 1, a government price control program is applied to an economy at an initial equilibrium position. This initial equilibrium position may or may not have been stable in the pre-control period.

The inflation control program is applied to lower the actual rate of inflation, so presumably the controlled inflation rate is set lower than the rate of change in the money supply $\pi_g < \frac{\dot{M}}{M}$, which initially equals the actual rate of inflation. The short-run adjustment path is the dynamic time path the system follows to the steady-state or long-run equilibrium position.

The phase diagram, in $r$-$\rho$ space, are shown in Figures 3.1 - 3.7. The arrows represent the directional movement about the new equilibrium point.[15] the solid lines are the initial curves and

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[15] See Footnote (10) in Chapter II for an explanation on the derivation of the arrows.

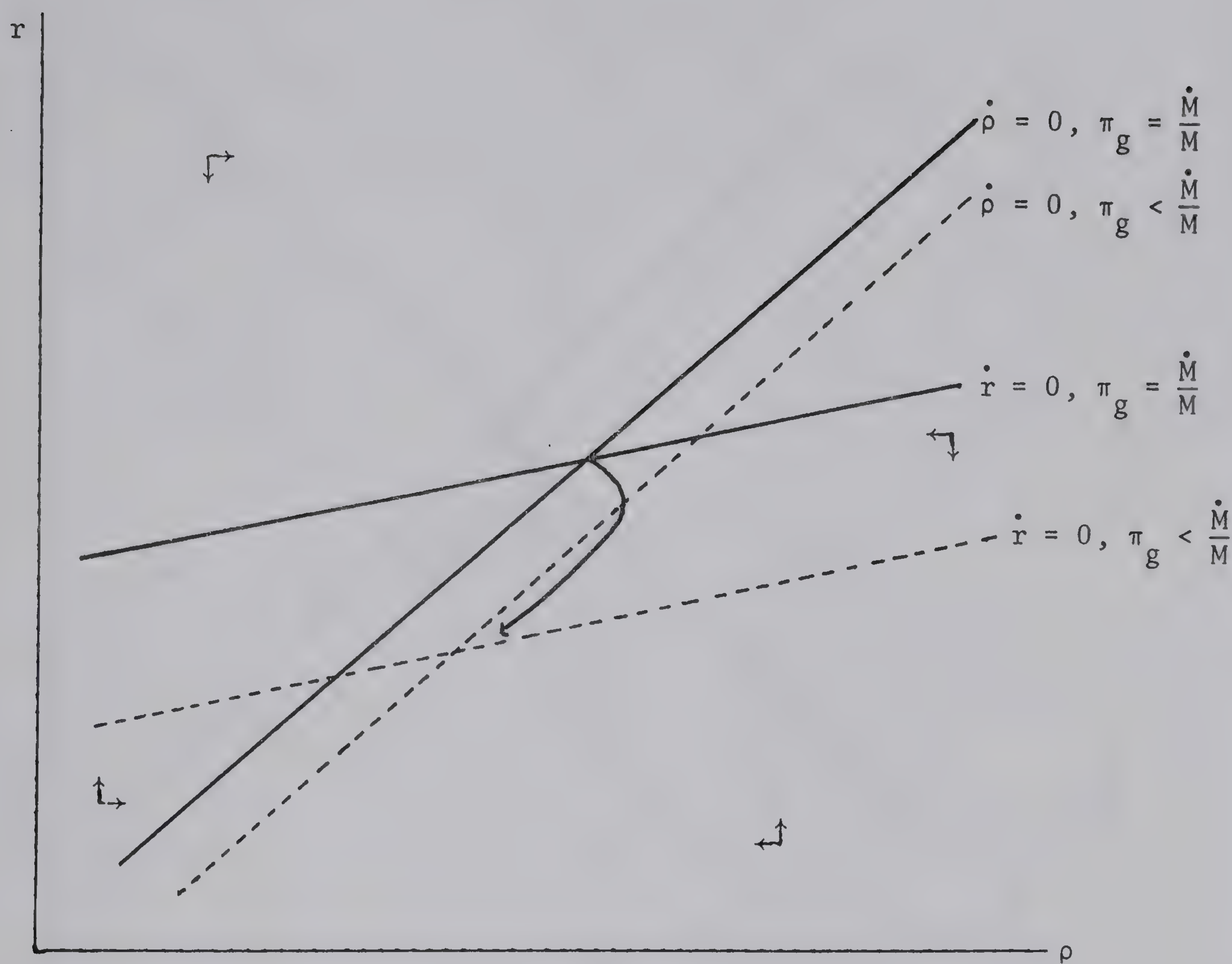


Figure 3.1

Phase Diagram for $\pi_g < \frac{\dot M}{M}$

$$\left.\frac{dr}{d\rho}\right|_{\dot\rho=0} > \left.\frac{dr}{d\rho}\right|_{\dot r=0} > 0$$

$$1 + \varepsilon - \mu\delta < 0$$

$$\lambda\beta < 1 + \varepsilon + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda)$$

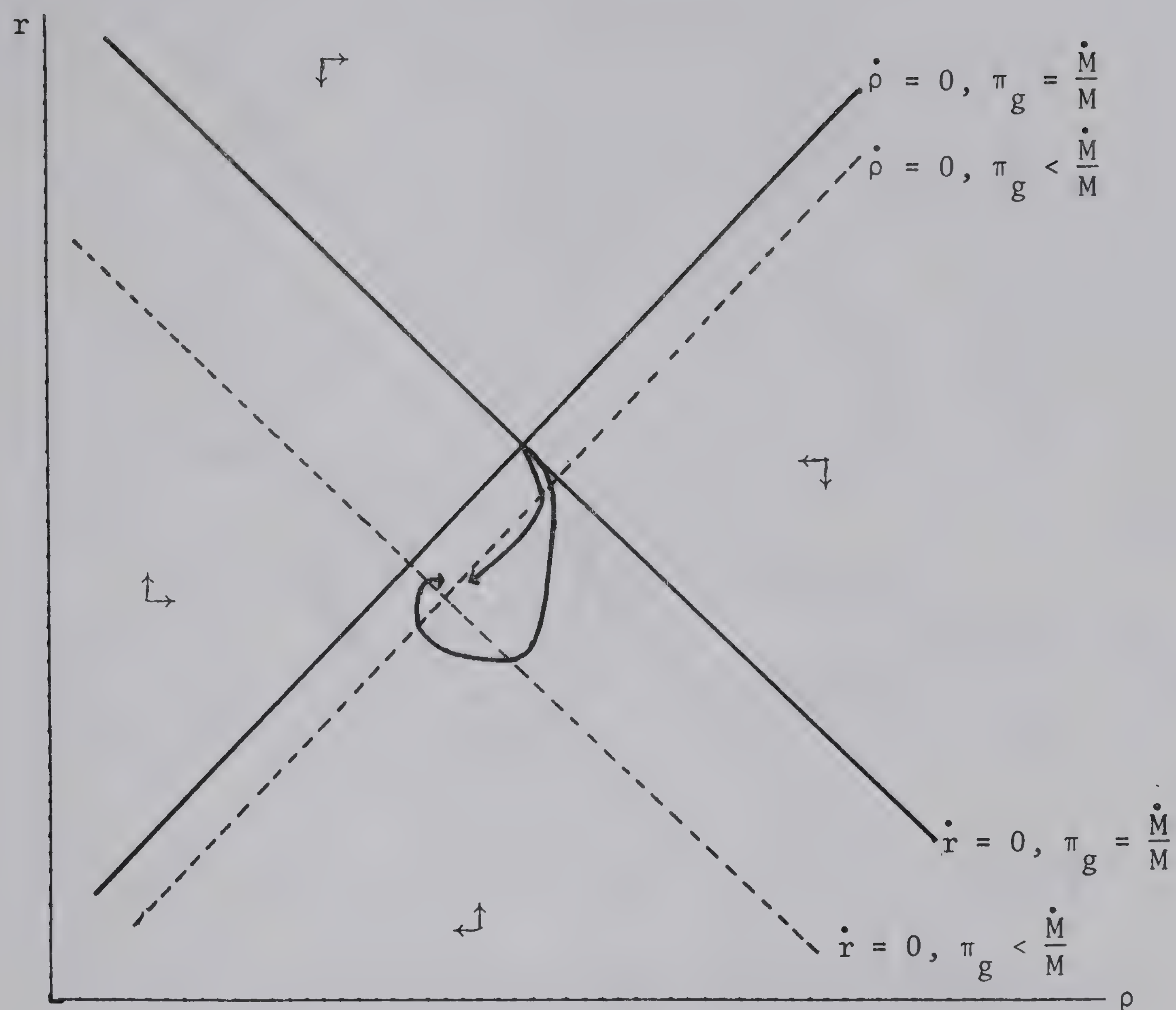


Figure 3.2

Phase Diagram for $\pi_g < \frac{\dot{M}}{M}$

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} < 0, \quad \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} > 0$$

$$1 + \varepsilon - \mu\delta < 0$$

$$\lambda\beta < 1 + \varepsilon + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda)$$

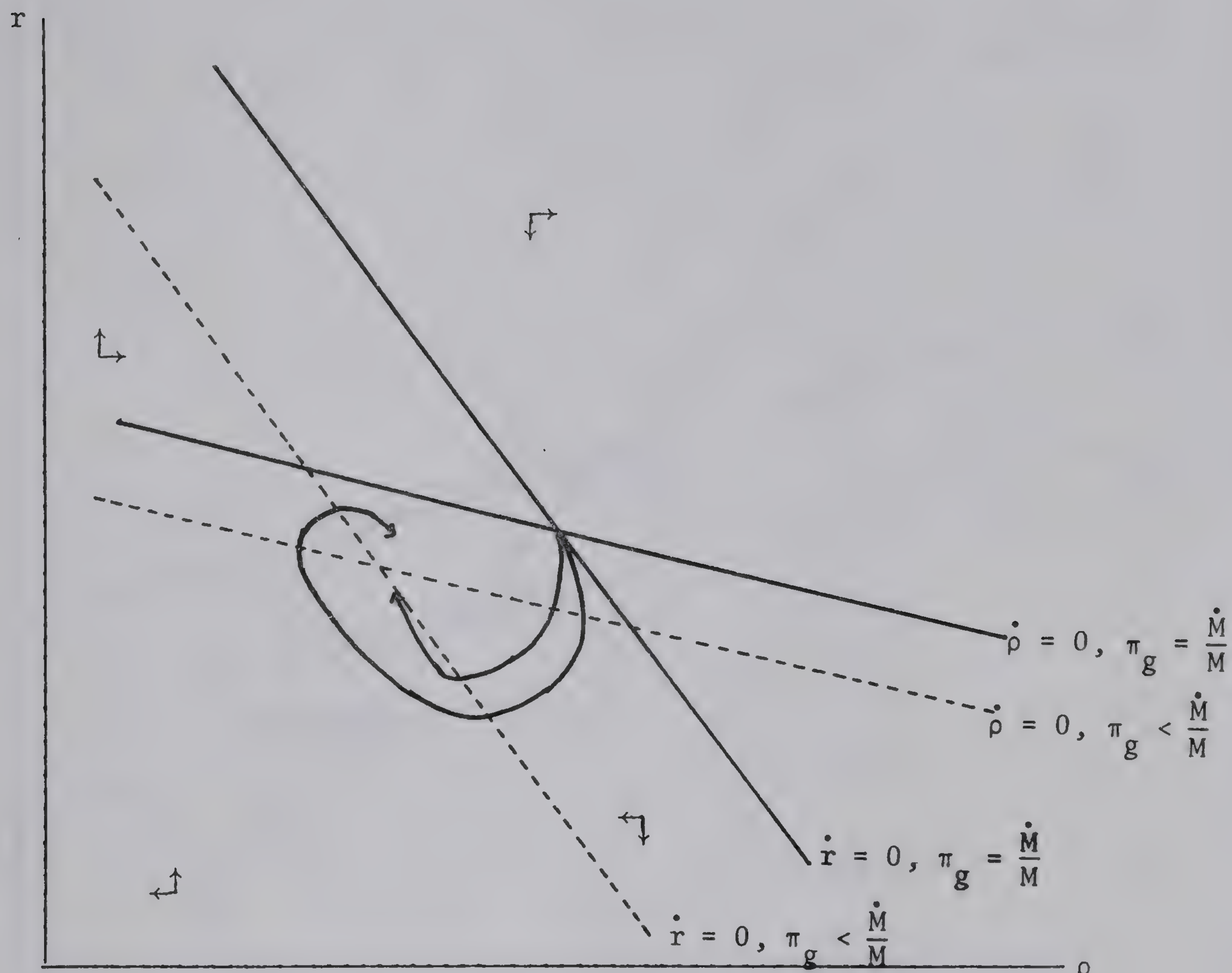


Figure 3.3

Phase Diagram for $\pi_g < \frac{\dot{M}}{M}$

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} < \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} < 0$$

$$1 + \varepsilon - \mu\delta < 0$$

$$\lambda\beta < 1 + \varepsilon + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda)^{\dagger}$$

†There is a possibility of the trace having a positive value, i.e., $\lambda\beta > 1 + \varepsilon + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda)$. The unstable movement away from the equilibrium position is not shown in the diagram.

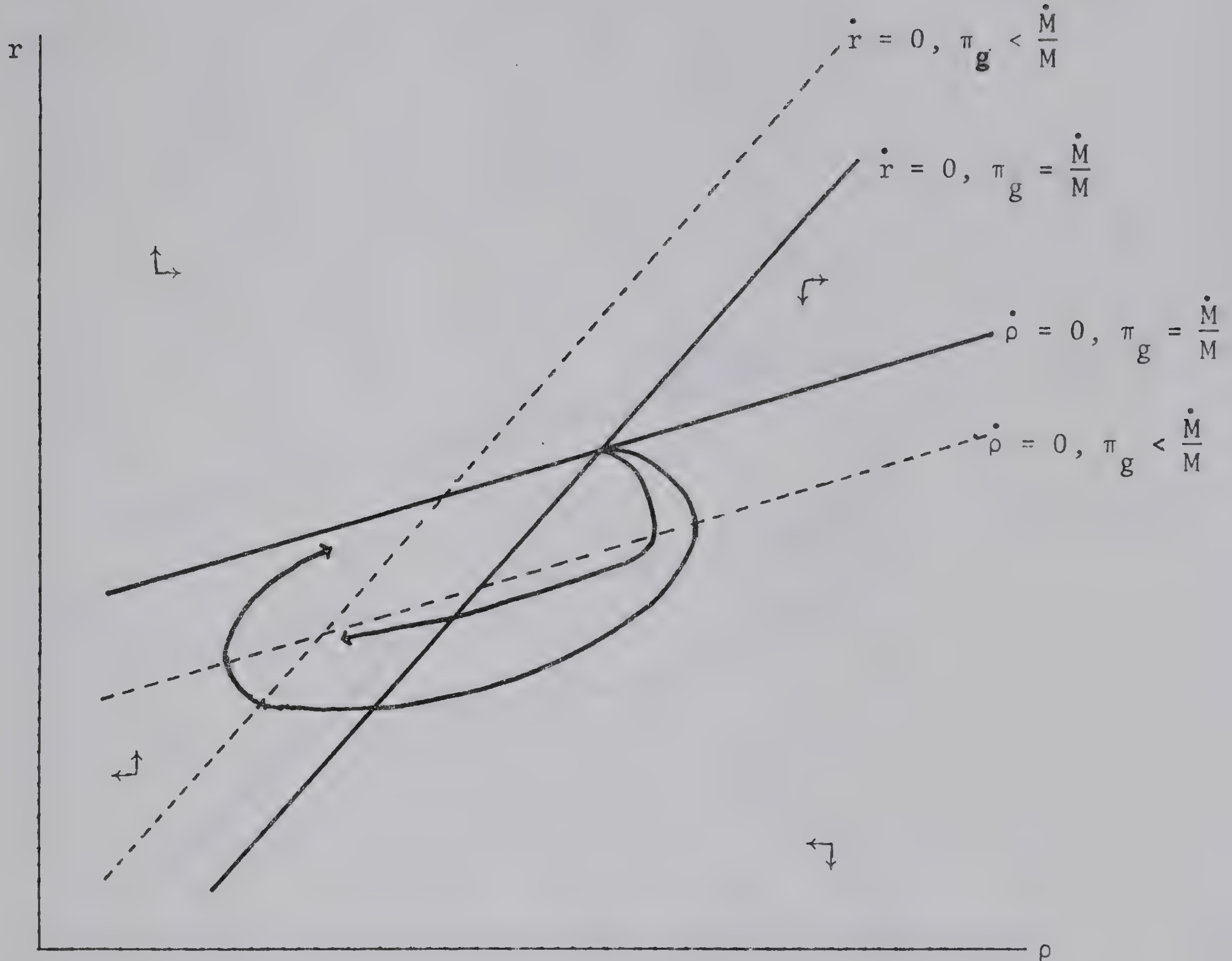


Figure 3.4

Phase Diagram for $\pi_g < \frac{\dot{M}}{M}$

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} > \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} > 0$$

$$1 + \varepsilon - \mu\delta > 0, \quad \varepsilon - \mu\delta < 0$$

$$\lambda\beta < 1 + \varepsilon + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda)^{\dagger}$$

†Note: There is a possibility of the trace having a positive value, i.e., $\lambda\beta > 1 + \varepsilon + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda)$. The unstable movement away from the equilibrium position is not shown in the diagram.

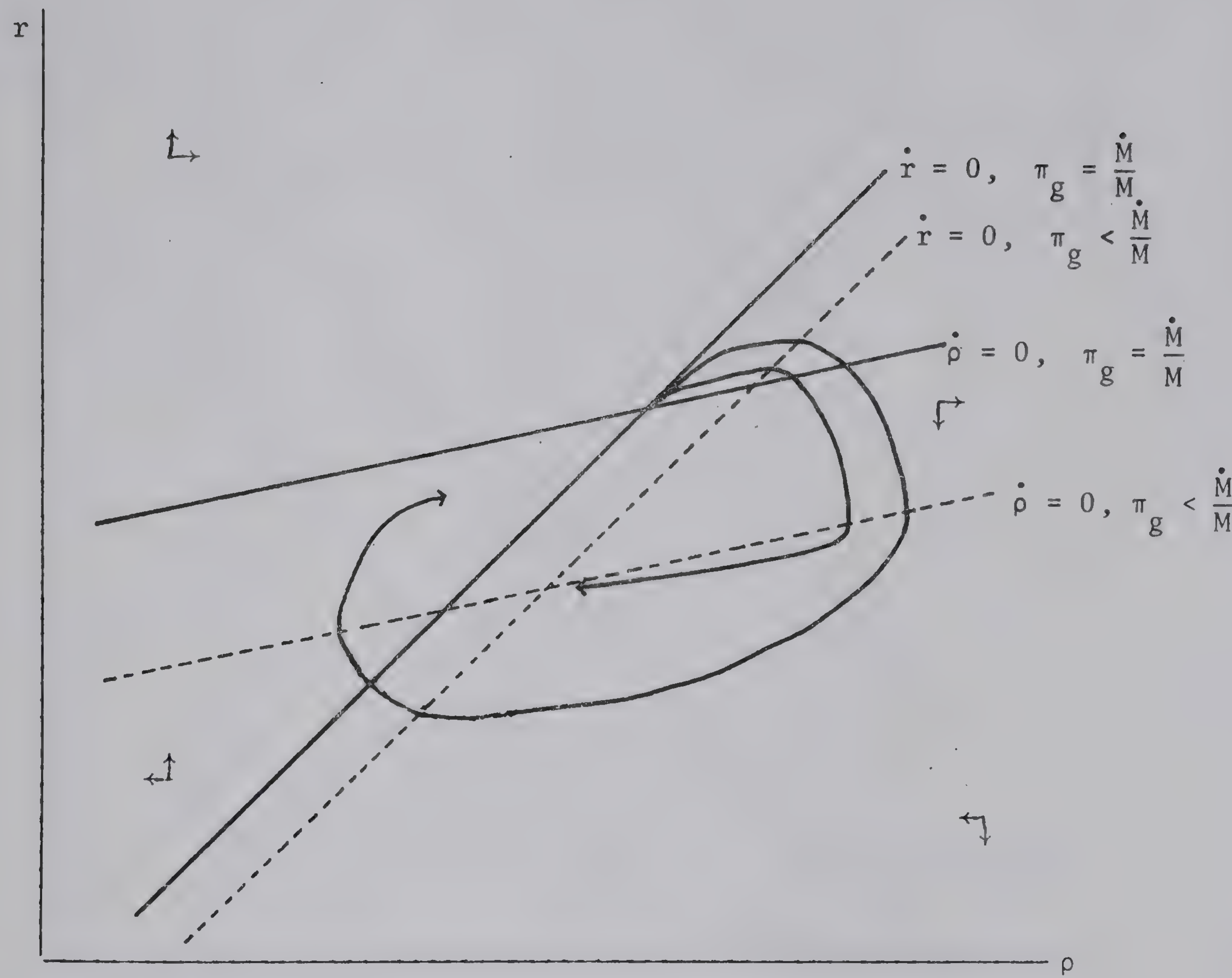


Figure 3.5

Phase Diagram for $\pi_g < \frac{\dot{M}}{M}$

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} > \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} > 0$$

$$\varepsilon - \mu\delta > 0$$

$$\lambda\beta < 1 + \varepsilon + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda)^{\dagger}$$

†Note: There is a possibility of the trace having a positive value, i.e., $\lambda\beta > 1 + \varepsilon + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda)$. The unstable movement away from the equilibrium position is not shown in the diagram.

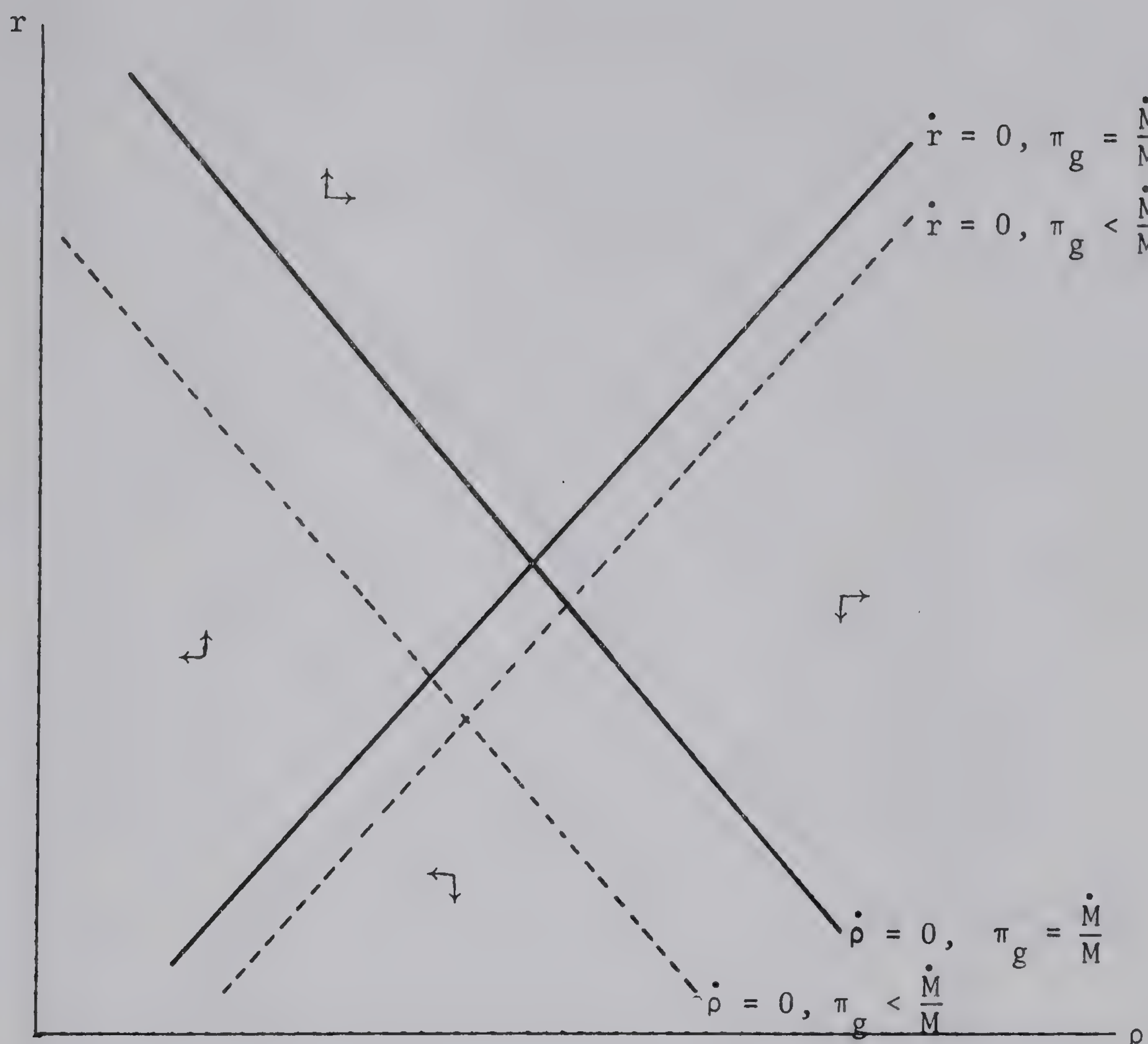


Figure 3.6

Phase Diagram for $\pi_g < \frac{\dot{M}}{M}$

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} > 0, \quad \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} < 0$$

$$1 + \varepsilon - \mu\delta > 0, \quad \varepsilon - \mu\delta < 0$$

$$\lambda\beta > 1 + \varepsilon + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda)$$

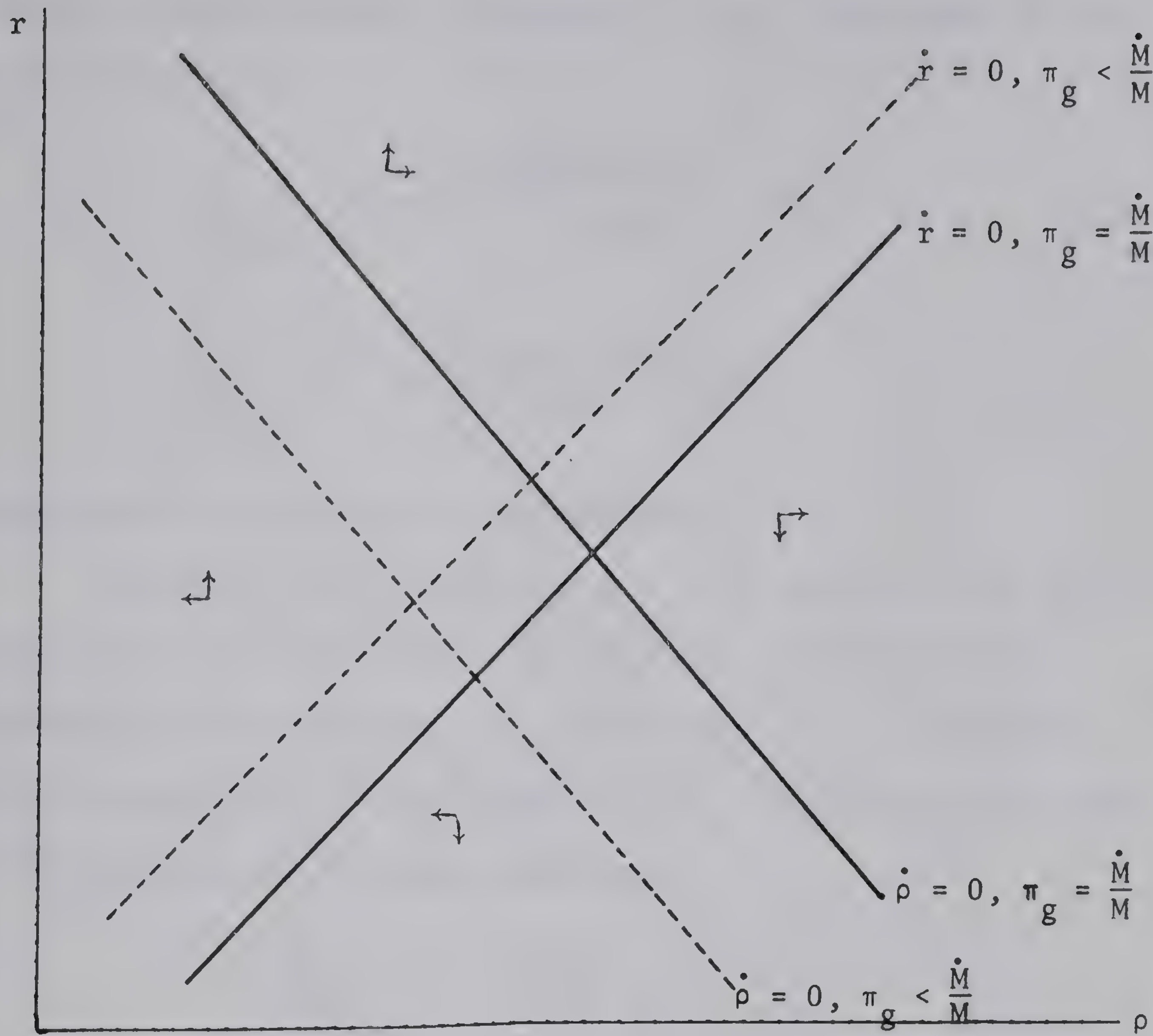


Figure 3.7

Phase Diagram for $\pi_g < \frac{\dot{M}}{M}$

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=o} > 0, \quad \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=o} < 0$$

$$\varepsilon - \mu\delta > 0$$

$$\lambda\beta > 1 + \varepsilon + \lambda\delta + \frac{\gamma\delta}{\mu}(\mu+\lambda)$$

the dahsed lines are the new curves after the implementation of an inflation control program (a decrease in $\pi_g$). The slopes of the two curves are:

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=o} = 1 + \frac{\frac{\mu}{\gamma}(1+\varepsilon+\mu(\beta-\delta))}{1+\varepsilon-\mu\delta} \gtrless 0$$

$$\left.\frac{dr}{d\rho}\right|_{\dot{\rho}=o} = 1 + \frac{\frac{\mu}{\gamma}(1+\varepsilon-\lambda(\beta-\delta))}{1+\varepsilon+\lambda\delta} \gtrless 0 .$$

The slopes of both curves are indeterminate.

The effect of a change in $\pi_g$ is to shift both the $\dot{r} = 0$ curve and the $\dot{\rho} = 0$ curve. The direction of the shift is determined in the following way. By setting $\dot{r} = 0$ in Eq (3.4), and differentiating $r$ with respect to $\pi_g$, we can see that the $\dot{r} = 0$ curve may shift either direction:

$$\left.\frac{\partial r}{\partial \pi_g}\right|_{\dot{r}=o} = \frac{\varepsilon-\mu\delta}{1+\varepsilon-\mu\delta} \gtrless 0 .$$

The direction of the shift of the $\dot{r} = 0$ curve depends on the signs of $1+\varepsilon-\mu\delta$ and $\varepsilon-\mu\delta$. For the case where $\pi_g < \frac{\dot{M}}{M}$, as shown in Figures 3.1 - 3.7, the $\dot{r} = 0$ curves shifts down when $\left.\frac{\partial r}{\partial \pi_g}\right|_{\dot{r}=o} > 0$, and shifts up when $\left.\frac{\partial r}{\partial \pi_g}\right|_{\dot{r}=o} < 0$, for a decrease in $\pi_g$.

In the same manner, setting $\dot{\rho} = 0$ in Eq (3.5) and differentiating $r$ with respect to $\pi_g$ yields the following expression:

$$\left.\frac{\partial r}{\partial \pi_g}\right|_{\dot{\rho}=o} = \frac{\varepsilon+\lambda\delta}{1+\varepsilon+\lambda\delta} > 0 .$$

This means that for a decrease in $\pi_g$, the $\dot{\rho} = 0$ curve shifts down in r-$\rho$ space as shown in Figures 3.1 - 3.7.

The relative sizes of the two shifts are similar to the cases cited in Model 1. The magnitudes of the shifts are examined in two parts below.

a. Case: $1 + \varepsilon - \mu\delta < 0$

In this case, the $\dot{r} = 0$ curve shifts more than the $\dot{\rho} = 0$ curve. Suppose this is not true, then:

$$\left.\frac{\partial r}{\partial \pi_g}\right|_{\dot{r}=0} \leq \left.\frac{\partial r}{\partial \pi_g}\right|_{\dot{\rho}=0} .$$

$$\text{i.e.,} \qquad \frac{\varepsilon-\mu\delta}{1+\varepsilon-\mu\delta} \leq \frac{\varepsilon+\lambda\delta}{1+\varepsilon+\lambda\delta} .$$

This simplifies to:

$$-(\mu+\lambda) \geq 0 .$$

Given the restrictions on the parameters, which are both positive, $\mu+\lambda > 0$ must hold. Therefore, the $\dot{r} = 0$ curve must shift more than the $\dot{\rho} = 0$ curve when $1+\varepsilon-\mu\delta < 0$.

b. Case: $1 + \varepsilon - \mu\delta > 0$

With this case, two possible results may occur.

i. $\varepsilon - \mu\delta < 0$

When $\varepsilon-\mu\delta < 0$, the $\dot{r} = 0$ curve shifts up for a negative change in $\pi_g$. Suppose the $\dot{\rho} = 0$ curve shifts less than the $\dot{r} = 0$ curve. Then:

$$\left.\frac{-\partial r}{\partial \pi_g}\right|_{\dot{r}=0} > \left.\frac{\partial r}{\partial \pi_g}\right|_{\dot{\rho}=0} ,$$

i.e., $$\frac{\mu\delta-\varepsilon}{1+\varepsilon-\mu\delta} > \frac{\varepsilon+\lambda\delta}{1+\varepsilon+\lambda\delta} .$$

This simplifies to:

$$-2\varepsilon(1+\varepsilon) + \delta(1+2\varepsilon)(\mu-\lambda) + 2\mu\lambda\delta^2 > 0 .$$

Given the restrictions on the parameters, $-\varepsilon(1+\varepsilon)+\delta(1+2\varepsilon)(\mu-\lambda)+2\mu\lambda\delta^2 \gtrless 0$. Therefore, the $\dot{r} = 0$ curve may or may not shift more than the $\dot{\rho} = 0$ curve when $1+\varepsilon-\mu\delta > 0$ and $\varepsilon-\mu\delta < 0$.

ii. $\varepsilon - \mu\delta > 0$

In this case, the $\dot{\rho} = 0$ curve shifts more than the $\dot{r} = 0$ curve. Suppose this is not true, then:

$$\left.\frac{\partial r}{\partial \pi_g}\right|_{\dot{r}=0} \geq \left.\frac{\partial r}{\partial \pi_g}\right|_{\dot{\rho}=0} ,$$

i.e., $$\frac{\varepsilon-\mu\delta}{1+\varepsilon-\mu\delta} \geq \frac{\varepsilon+\lambda\delta}{1+\varepsilon+\lambda\delta} .$$

This simplifies to:

$$\mu + \lambda \leq 0 .$$

Given the restrictions on the parameters, $\mu+\lambda > 0$ must hold. Therefore, the $\dot{\rho} = 0$ curve shifts more than the $\dot{r} = 0$ curve when $\varepsilon-\mu\delta > 0$.

The seven cases shown in Figures 3.1 - 3.7 are the only ones possible.(16) The cases are identified in the following manner. By

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(16) The special case when $1+\varepsilon-\mu\delta = 0$ is briefly examined in Appendix II-C.

subtracting the slopes of the two curves we can examine the magnitude of their differences:

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} - \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} = \frac{\mu\beta(1+\varepsilon)(\mu+\lambda)}{\gamma(1+\varepsilon+\lambda\delta)(1+\varepsilon-\mu\delta)} \cdot$$

When $1+\varepsilon-\mu\delta < 0$, the slope of the $\dot{r} = 0$ curve is less than the slope of the $\dot{\rho} = 0$ curve, $\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} < \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0}$, resulting in the three following cases:

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} > 0, \quad \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} > 0, \qquad \text{(Figure 3.1)}$$

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} < 0, \quad \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} > 0, \qquad \text{(Figure 3.2)}$$

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} < 0, \quad \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} < 0. \qquad \text{(Figure 3.3)}$$

Four possible cases exist when $\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} > \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0}$. Two cases are the result of $1+\varepsilon-\mu\delta > 0$, but $\varepsilon-\mu\delta < 0$:

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} > 0, \quad \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} > 0, \qquad \text{(Figure 3.4)}$$

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} > 0, \quad \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} < 0. \qquad \text{(Figure 3.6)}$$

The final cases are those when $\varepsilon-\mu\delta > 0$:

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} > 0, \quad \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} > 0, \qquad \text{(Figure 3.5)}$$

$$\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} > 0, \qquad \left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} < 0 . \qquad \text{(Figure 3.7)}$$

Upon inspection of the slope of the $\dot{r} = 0$ curve, we can easily see that when $1+\varepsilon-\mu\delta$ is positive, the $\dot{r} = 0$ curve cannot be downward sloping. Therefore, these are the only four possible cases when $1+\varepsilon-\mu\delta > 0$.

Starting from a pre-control equilibrium position where $\frac{\dot{M}}{M} = \pi$, the application of a government inflation control program where $\pi_g < \frac{\dot{M}}{M}$ causes the $\dot{r} = 0$ and $\dot{\rho} = 0$ curves to shift in the manner stated above. Out of the seven cases, five are either stable or assumed to be stable (Figures 3.1 - 3.5) and two are unstable (Figures 3.6 - 3.7). These unstable cases are not discussed here. All five stable cases may approach the equilibrium in an asymptotic manner, while only four cases allow for the possibility of a cyclical approach (Figures 3.2 - 3.5). In all five cases the movement is in a clockwise direction around the new equilibrium position.

Typical time paths for both interest rates, output, and both inflation rates are shown in Figures 3.8 - 3.10. The short-run time paths presented in Figures 3.8 and 3.9 are identical to those in Model 1 in Figures 2.6 and 2.7. Therefore, these diagrams are not discussed here.

Only Figure 3.5 differs from the phase diagrams of Model 1. The corresponding time paths are shown in Figure 3.10. This case differs from the other four in that the nominal interest rate increases initially and then declines. Since the nominal interest

rate is rising initially, the actual and expected inflation rates are not falling as fast initially as they do in the other four cases. Aside from this difference, the movements are the same in all five typical cases.

3. <u>The Effect of Inflation Controls When</u> $\pi_g \geq \frac{\dot{M}}{M}$

When the economy is at its long-run equilibrium position and an inflation control program is applied such that the controlled inflation rate is set equal to the actual inflation rate, the economy does not move from its equilibrium position and the control is ineffective. In this case, $P = P_g$ and the model reduces to Model 1.

When the controlled inflation rate is set higher than the actual inflation rate, and $\delta = \varepsilon = 0$, the control is ineffective. Another possibility exists when $\pi_g > \frac{\dot{M}}{M}$, $\delta > 0$ and $\varepsilon > 0$. This situation was discussed in the previous chapter in Section B-3. The results are the opposite of those shown in Figures 3.1 - 3.10. Initially both inflation rates and output rise while the real interest rate falls. The nominal interest rate rises in the four cases corresponding to Figures 3.1 - 3.4. In the case corresponding to Figure 3.5 the nominal interest rate initially falls, then rises.

C. THE LONG-RUN EQUILIBRIUM POSITION

The time path movement is towards the long-run steady-state equilibrium position, given that local stability conditions hold. The equilibrium positions for the endogenous variables are obtained in the same manner as in Model 1, in Chapter II, Section C, by using the corresponding equations in Model 2. The resulting equilibrium values

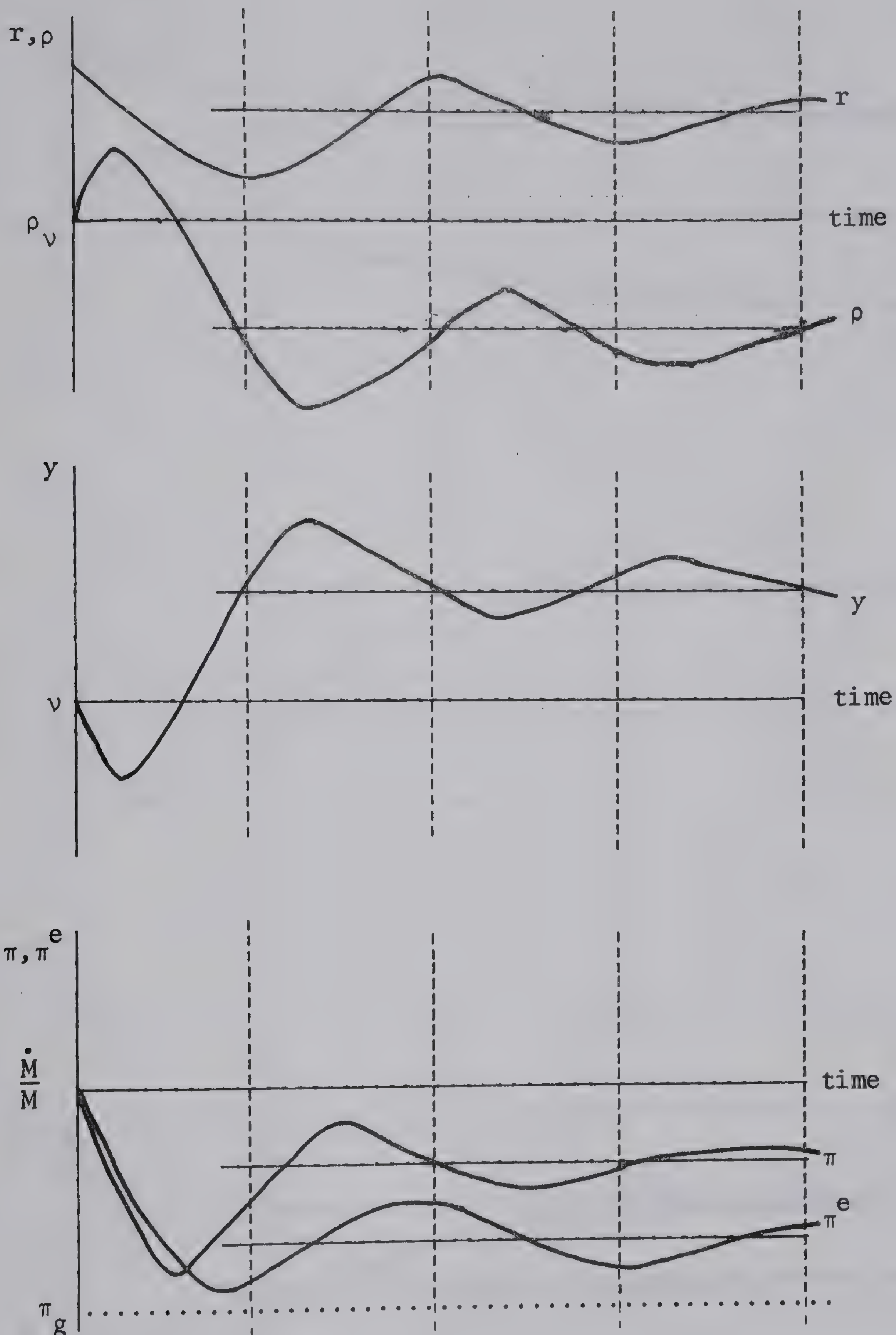


Figure 3.8

Time Paths for $\pi_g < \frac{\dot{M}}{M}$ : Cyclical Case†

†Note: These time paths, corresponding to Figures 3.1 - 3.4, are for a typical case only. Other oscillatory paths may occur.

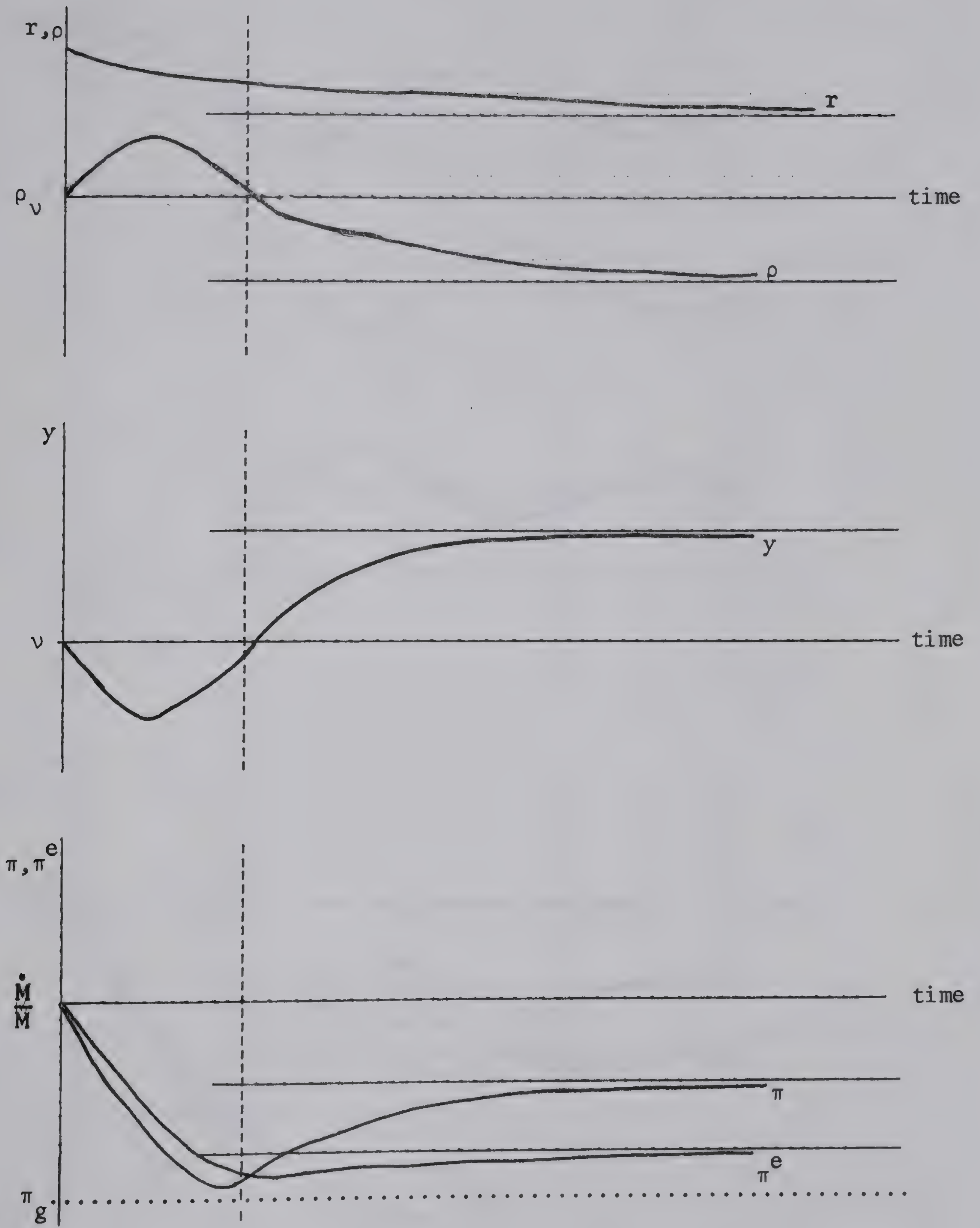


Figure 3.9

Time Paths for $\pi_g < \frac{\dot{M}}{M}$ : Asymptotic Cases†

†Note: These time paths, corresponding to the asymptotic approach in Figures 3.1 - 3.4 are for a typical case only. Other paths may occur.

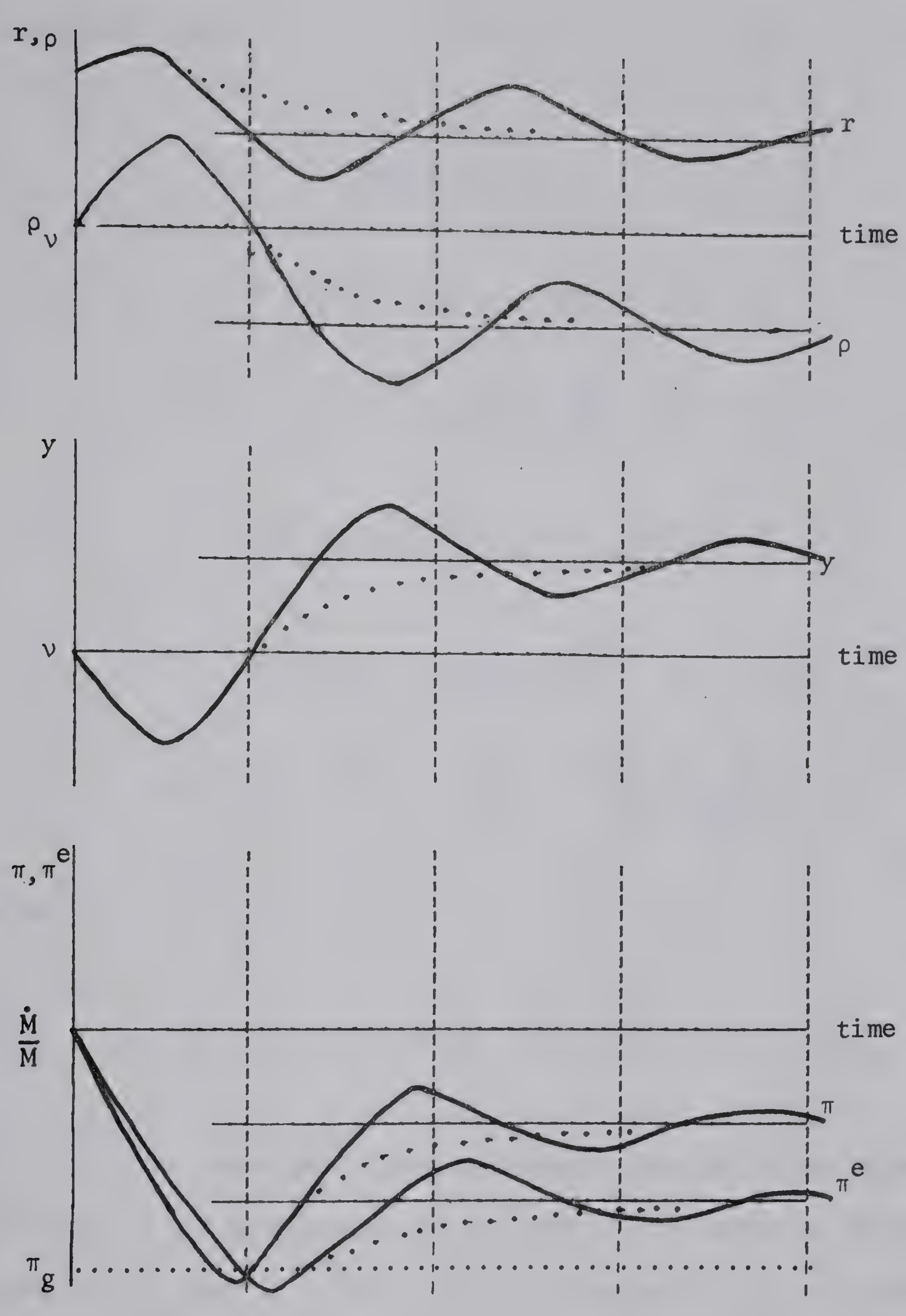


Figure 3.10

Time Paths for $\pi_g < \frac{\dot{M}}{M}$ : Special Case†

†Note: These time paths, corresponding to Figure 3.5, are for typical cases only. Other oscillatory or asymptotic paths may occur. The dotted curves show the asymptotic paths.

are the following:

$$\pi = \frac{1}{1+\varepsilon}\frac{\dot{M}}{M} + \frac{\varepsilon}{1+\varepsilon}\pi_g$$

$$\pi^e = \frac{1}{1+\varepsilon}\frac{\dot{M}}{M} + \frac{\varepsilon}{1+\varepsilon}\pi_g - \frac{\delta}{\beta(1+\varepsilon)}\left(\frac{\dot{M}}{M} - \pi_g\right)$$

$$\rho = \rho_\nu - \frac{\gamma\delta}{\beta\mu(1+\varepsilon)}\left(\frac{\dot{M}}{M} - \pi_g\right)$$

$$r = \rho_\nu + \frac{1}{1+\varepsilon}\frac{\dot{M}}{M} + \frac{\varepsilon}{1+\varepsilon}\pi_g - \frac{\delta(\gamma+\mu)}{\beta\mu(1+\varepsilon)}\left(\frac{\dot{M}}{M} - \pi_g\right)$$

$$\ell n\ y = \ell n\ \nu + \frac{\delta\gamma}{\beta(1+\varepsilon)}\left(\frac{\dot{M}}{M} - \pi_g\right)$$

$$\ell n\left(\frac{M}{P}\right) = \ell n\left(\frac{M}{P}\right)_2 - \left[\frac{\lambda\delta}{\beta(1+\varepsilon)} + \frac{\delta\gamma(\lambda-1)}{\beta\mu(1+\varepsilon)}\right]\left(\frac{\dot{M}}{M} - \pi_g\right)$$

where

$$\ell n\left(\frac{M}{P}\right)_2 = \phi - \lambda\left(\rho_\nu + \frac{\dot{M}}{M}\right) + \ell n\ \nu + \varepsilon(\ell n\ P - \ell n\ P_g) \quad .$$

The exact positions of the equilibrium values depend upon the size of the parameters $\beta$, $\delta$ and $\varepsilon$, as well as the relative sizes of $\frac{\dot{M}}{M}$ and $\pi_g$. The larger the value of $\beta$, the closer all variables (except the actual inflation rate) are to their pre-control values. The larger the value of $\delta$, the further the endogenous values are from their pre-control values. The larger the value of $\varepsilon$, the closer the actual and expected rates of inflation are to the controlled rate of inflation. Also, large values of $\varepsilon$ cause the nominal interest

rate to be closer to the sum of the natural rate of interest and the government controlled inflation rate. This causes the level of output to approach its natural level and therefore, the real interest rate to its natural rate. Note, it is most likely the case that large values of $\varepsilon$ are accompanied by large values of $\delta$. The reason for this is that the greater the effect the control program has on the demand for money, the greater the effect the program probably has on expectations.

1. Case: $\pi_g < \frac{\dot{M}}{M}$

In this case, the government sets the controlled inflation rate below the rate of change in the money supply. The equilibrium position corresponds to the new equilibrium positions shown in Figures 3.1 - 3.5. The equilibrium value of the actual inflation rate is below its pre-control value. It is positioned somewhere between the rate of change in the money supply and the government controlled rate of inflation. The expected inflation rate has a value equal to the actual inflation rate if $\delta = 0$. Otherwise the expected inflation rate is below the actual inflation rate but above the government controlled inflation rate. Both the real and nominal interest rates remain below their pre-control rates. The level of output has a value above its natural level and the level of real money balances are above their pre-control equilibrium level.

2. Case: $\pi_g = \frac{\dot{M}}{M}$

When the government controlled inflation rate is set equal to the rate of change in the money supply, the long-run equilibrium position is the same as its uncontrolled equilibrium position. This position has been discussed in Chapter II.

3. Case: $\pi_g > \frac{\dot{M}}{M}$

In this case, the position may remain at its pre-control equilibrium position as the control has no effect on expectations, and the demand for real money balances is unaffected by the ratio $\frac{P}{P_g}$ . This is the case when $\varepsilon = \delta = 0$.

Another possibility exists where $\varepsilon > 0$, $\delta > 0$, and people use the control rate as a bargaining position. This is described in Model 1. If this occurs the actual inflation rate is above the rate of change in the money supply but below the controlled inflation rate. Expectations remain above the actual inflation rate. The level of output and the level of real money balances are below their pre-control levels while both interest rates are above their uncontrolled rates.

D. THE POST-CONTROL PERIOD

Often these long-run equilibrium positions are never attained as the control program is removed before they are reached. If the control program is left on indefinitely, the long-run positions will differ from the post-control equilibrium positions. The effect of removing the control program for both a stable and unstable post-control

economy are examined here. Emphasis is on the case where $\pi_g < \frac{\dot{M}}{M}$.

1. <u>Stable Post-Control Conditions After</u> $\pi_g < \frac{\dot{M}}{M}$ <u>Controls</u>

When the inflation control program is removed, the long-run equilibrium position returns again to its pre-control position. If this equilibrium is stable, the economy adjusts towards this position. If the post-control stability condition is violated, $\lambda\beta \geq 1$, the economy moves away from the equilibrium position.

The stable post-control, long-run equilibrium is shown in Figure 3.11. The possible regions, in the stable case, represented by numbers one through four are identical to the four regions discussed in Model 1. These four regions are not discussed here. As before, the initial starting position may fall anywhere within the shaded area.

Regions five and six are areas where initial positions may occur if the control period is represented by Figures 3.5. These positions may occur if the program is removed while it is in its early stages or late stages if a cyclical movement exists. Both positions correspond to interest rates being above their long-run equilibrium values while the level of output and both inflation rates are below their equilibrium values. The level of real money balances is higher than its equilibrium level. In region five, both interest rates and real money balances are rising while output, and both the actual and expected inflation rates are falling. In region six, the nominal interest rate is falling along with output and both inflation rates while the real interest rate and real cash balances are both increasing.

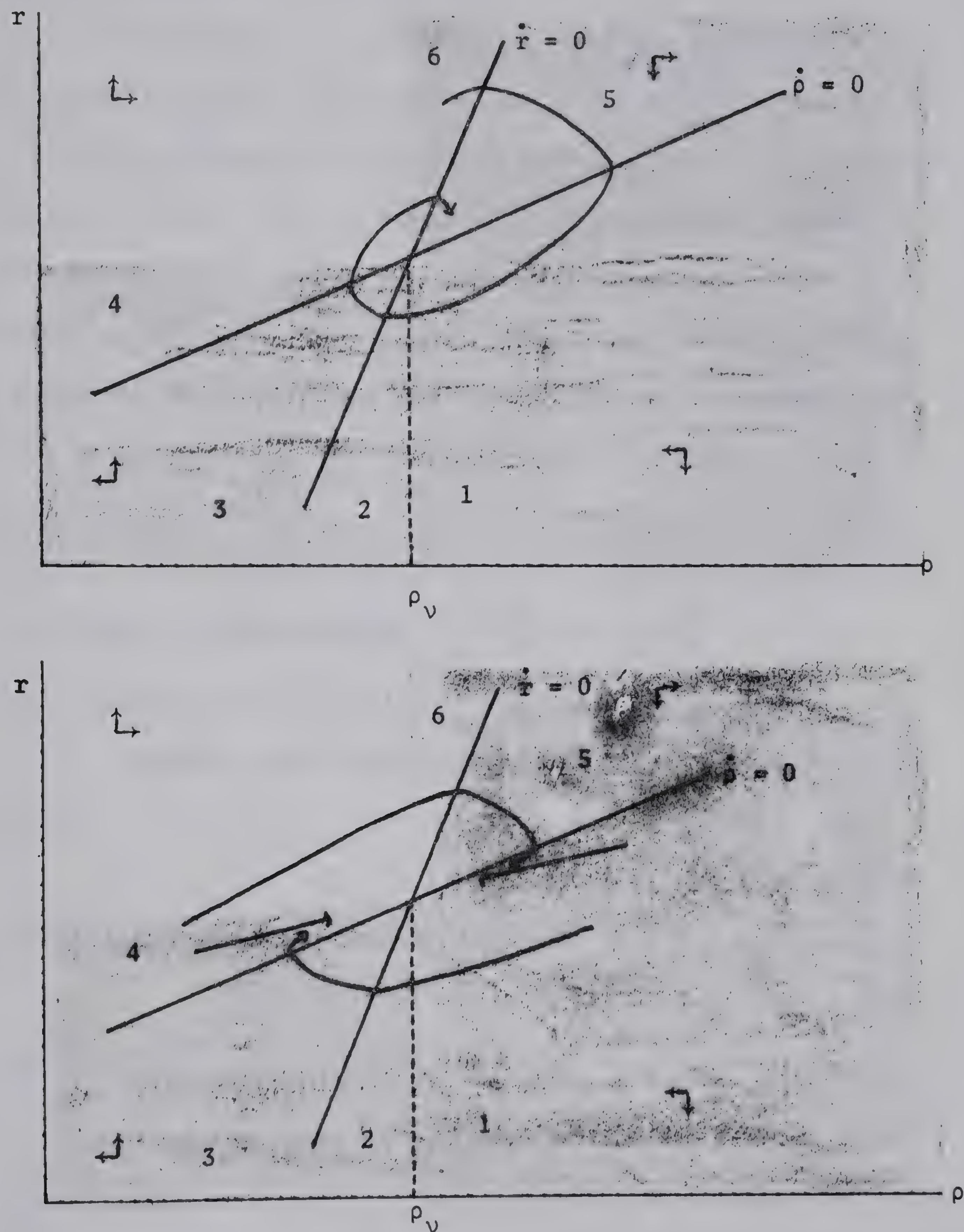


Figure 3.11

Phase Diagram for a Stable Post-Control Equilibrium $\lambda\beta < 1^{\dagger}$

[†]Note: The non-dominant separatrix has been omitted from the bottom diagram for clarity.

2. Unstable Post-Control Conditions After $\pi_g < \frac{\dot{M}}{M}$ Controls

If the equilibrium is unstable, only one new possible initial position exists from those shown in Figure 2.9 of Model 1. This is shown in Figure 3.12 where both interest rates are above their equilibrium values. It is an explosive time path away from the equilibrium position. Again, the time path may be asymptotic or cyclical. Initially both the real and nominal interest rates rise while the level of output and both the actual and expected rates of inflation fall. The level of real money balances initially decline also.

3. The Removal of Controls When $\pi_g = \frac{\dot{M}}{M}$ or $\pi_g > \frac{\dot{M}}{M}$

Removing controls when $\pi_g = \frac{\dot{M}}{M}$ or $\pi_g > \frac{\dot{M}}{M}$ and $\delta = \varepsilon = 0$, does not affect the economy at all as the program was ineffective.

When $\pi_g > \frac{\dot{M}}{M}$ and $\delta > 0$, $\varepsilon > 0$, the time paths will be the mirror image of those when $\pi_g < \frac{\dot{M}}{M}$.

E. SUMMARY

The implementation of an inflation control program, where $\pi_g < \frac{\dot{M}}{M}$, causes the economy to follow a cyclical or asymptotic path to a new equilibrium position if the economy is stable. In the short-run, all the endogenous variables may fluctuate about their long-run equilibrium positions. Once the long-run equilibrium position is attained, the level of output and real money balances

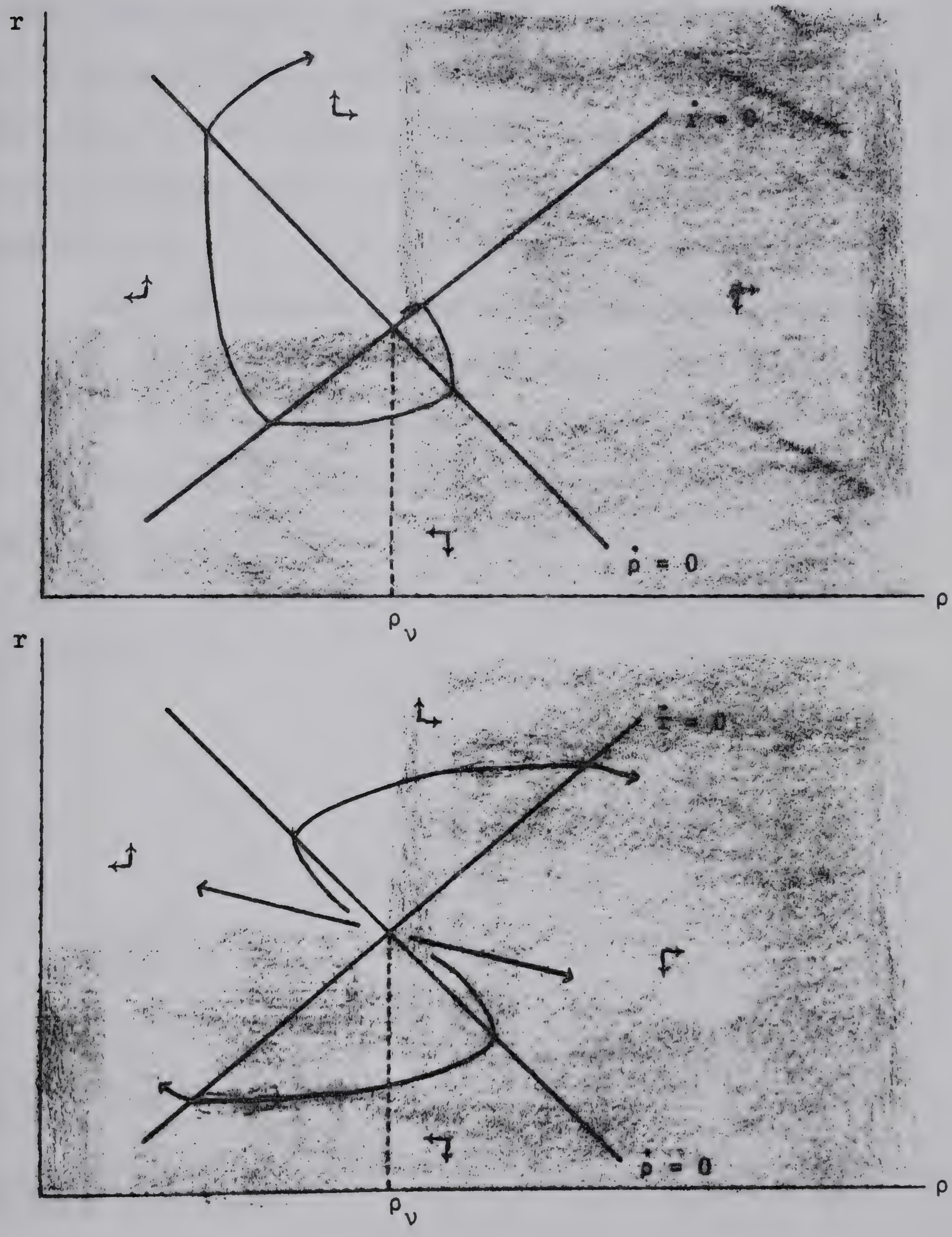


Figure 3.12

Phase Diagram for an Unstable Post-Control Equilibrium $\lambda\beta \geq 1$†

†Note: The non-dominant separatrix has been omitted from the bottom diagram for clarity.

have higher values than their pre-control equilibrium values. Both interest rates as well as the actual and expected infaltion rates have lower equilibrium values than in the pre-control period. Even though the actual and expected inflation rates are below their pre-control values, both rates are above the government controlled inflation rate.

When the inflation control program is removed, the long-run equilibrium position returns to its pre-control equilibrium. The economy may or may not approach the post-control, long-run equilibrium, depending on whether or not it is stable. Therefore, even though the control program does affect the inflation rate, it will only be a temporary effect unless the control remains on permanently.

# CHAPTER IV

## SUMMARY, CONCLUSIONS AND EXTENSIONS

Two models of inflation control have been developed in this thesis, Model 2 being an extension of Model 1. The dynamic time paths, the long-run equilibrium and the post-control period have all been examined in detail. In this chapter, the models are briefly summarized. The conclusions drawn from the models are discussed next. A few possible extensions which could be incorporated into the models conclude the chapter.

### A. SUMMARY OF THE MODELS

#### 1. Model 1

The inflation control program is imposed in a way such that it only affects inflationary expectations. People observe the difference between the actual and controlled rate of inflation. If the actual rate is above the controlled rate, people expect the actual rate to fall. This causes a downward change in expectations of inflation. The more effect the control program has on expectations, the more the long-run equilibrium position associated with the control program differs from the pre-control equilibrium. The long-run equilibrium is characterized by higher levels of output and real money balances, and lower rates of nominal and real interest rates. Inflationary expectations are lower in the long-run, but the actual rate of

inflation still equals the rate of change in the money supply. The short-run time path towards the equilibrium is a period where fluctuating values of all the endogenous variables may occur.

The removal of the inflation control program causes the long-run equilibrium position to return to its pre-control position. The economy moves back towards this position if the post-control equilibrium is stable, or follows an explosive path away from the equilibrium if the stability conditions are not satisfied. Therefore, the effect of the inflation control program on the endogenous variables is temporary, unless the control program is left on permanently.

2. Model 2

Model 2 is an extension of the first model. Since the control program does not affect the long-run actual inflation rate in Model 1, people would probably place less importance on the control program over time. This would cause the equilibrium position to move back to its pre-control position. Model 2 proposes that along with the effect on inflationary expectations, the control program also affects the demand for real money balances. The imposition of the program causes people to expect the nominal interest rate to fall, and real output to rise thereby altering the demand for real balances. The expected nominal interest rate and the expected real output are not easily observable. Therefore, we assume that an observable price ratio is used as an approximation for the effect these two variables have on the demand for real money balances. The price ratio used is the actual price level over the price level associated with the controlled

inflation rate at each point in time. The more people believe the program to succeed, the more effect the price ratio has on the demand for real money. The result of this is that in the long-run, the actual inflation rate is lowered towards the government controlled inflation rate. The long-run values of the other endogenous variables have moved in the same direction as they did in Model 1. Both the short-run and the post-control movements of all the endogenous variables are similar for both models.

## B. IMPORTANT CONCLUSIONS

Three main conclusions can be drawn from this thesis. The first conclusion is that the control program is temporary in nature. The result of the control having a temporary effect coincides with Possen's [32] result and a discussion put forth by Lipsey [25]. The other two conclusions have to do specifically with the equations employed in the models. The second conclusion is that the control program would have to permanently alter the structural equations to cause the post-control equilibrium to differ from the pre-control equilibrium. The final conclusion is that the control program has to alter the demand for money function in order to change the inflation rate. These conclusions are discussed in detail below.

The first observation one makes of the inflation control programs is that the effects on the endogenous variables tend to be temporary and not permanent. As soon as the control program is removed, the economy moves towards its uncontrolled equilibrium again, providing stability conditions are satisfied. Therefore, an inflation control program is not a good long-run policy, however, it may be a

good short-run policy. If a control program is imposed in conjunction with a contractionary monetary policy, the program may reduce some of the initial recessionary impact of the monetary policy. Lowering the growth rate of the money supply causes the level of output to fall initially (Taylor [38]). Inflationary expectations fall only after the actual inflation rate declines. Applying an inflation control program, where the controlled rate is less than the actual rate, causes expectations to start falling contemporaneously or before the actual inflation rate falls. Expectations fall because people expect the actual inflation rate will move towards the controlled rate. Therefore, applying an inflation control program along with a decrease in the money supply growth rate may lessen the initial fall in the level of output that would have occurred with the monetary policy alone.

Suppose the growth rate of the money supply is decreased and an inflation control program is imposed such that the controlled inflation rate equals the new, lower, money supply growth rate. The long-run equilibrium position is one where the level of output and the real interest rate are at their natural positions. The actual and expected inflation rates would both equal the growth rate of the money supply. The nominal interest rate would equal the real interest rate plus the actual inflation rate. This is the same long-run result that is obtained with a policy of decreasing the money supply growth rate without a control program. Therefore, removing the control program would not cause any movement away from the long-run equilibrium. Therefore, if the only intention of the control program is to lower the rate of inflation in the long-run, the program is not needed. However, the short-run effects of the program may make it

worthwhile to implement along with the monetary policy.

The second conclusion is that an inflation control program has to permanently change the structural equations of the model if the controls are to have an effect on the post-control equilibrium position. When the control program is imposed in our models, inflationary expectations and the demand for real money balances (in Model 2) equations are altered. Once the program is removed, these two equations once again become the same as in the pre-control period. The post-control model is identical to the pre-control model. Therefore, the control program cannot possibly have any permanent effect on the endogenous variables. In order for the program to affect the post-control equilibrium, the control program must alter the structural equations in the post-control model.

The final conclusion is that in our models, the equilibrium inflation rate will not differ from the growth rate of the money supply unless the control program alters the money demand function, as in Model 2. Since the long-run equilibrium position for the actual inflation rate is obtained by differentiating the LM function (explained on page 46), the control program can only affect the equilibrium inflation rate if control terms appear in the LM function and do not disappear upon differentiation. Since the control program does not alter the LM function in Model 1, the actual inflation rate equals the money supply growth rate in the long-run. By changing the demand for money function in Model 2, the equilibrium inflation rate may differ from the growth rate of the money supply.

## C. EXTENSIONS

The two models analyzed in this thesis are very simplified versions of a closed economy. A number of changes to the models could be made which may or may not alter the results of the control program. Two extensions are examined in this section. The first pertains to the conclusion that the controls are temporary in nature only. A cost-benefit analysis could be conducted to determine if the control program is worthwhile. The second extension consists of altering the Phillips curve equations during the control program similar to Possen [32] and Lipsey [24]. While these are the two extensions chosen to be examined here, other possibilities exist. Rational expectations could be employed instead of adaptive expectations if a stochastic model is used. Disequilibrium in the product and money markets could be incorporated into our models by changing the equilibrium function representing these markets into partial adjustment forms. In these cases, the change in the interest rate or the supply of output would adjust toward the difference between the demand and supply of money or output, depending upon the market. The models in this thesis maintained Taylor's assumption of a zero growth rate of output.[17] This assumption could be removed to allow for a non-zero growth rate of output. This thesis employs a semi-logarithmic demand for money function. Various money demand functional forms could be employed such as the linear and the constant elasticity of money demand functions of Yarrow's [43]. Numerous other changes are possible such as allowing for money illusion and the inclusion of a government budget constraint.

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[17] D. Taylor [38].

A cost-benefit analysis would enable one to weigh the benefits of an inflation control program against its costs to determine if the program is beneficial. The benefits of the control program can be seen by examining the time paths of the level of output and the actual inflation rate in Models 1 and 2. The level of output is raised above its natural level for an unknown period of time in the short-run, long-run, and post-control periods. Also, the rate of inflation is lowered for an indeterminate period of time in the short-run, long-run (in Model 2 only), and post-control periods.

The costs of the control program include the periods of time where the level of output is less than its natural level. Also, the inflation rate may rise above its pre-control equilibrium position in both the short-run and post-control periods. By weighing the benefits of the control program against its costs one could determine if the control program is a worthwhile policy even if its effects are of a temporary nature only.

The calculation of an inter-temporal loss function similar to the form Scarfe [35] uses in his analysis of monetary growth rates may be used for the cost-benefit analysis. The loss function would have expressions to measure the fluctuations of output from its natural level, the actual inflation rate from the money supply growth rate and the actual inflation rate from the controlled inflation rate. Specifically, the function could be of the form:

$$L = \int_{o}^{\infty} e^{-bt}\{f_1(t)[\nu-y(t)] + f_2(t)[\pi(t) - \frac{\dot{M}}{M}] + f_3(t)[\pi(t)-\pi_g]\}dt; \qquad b \geq 0$$

where $f_1(t)$, $f_2(t)$ and $f_3(t)$ are weighting functions and b is a discount factor. The terms $y(t)$ and $\pi(t)$ represent the values of

output and the inflation rate at time t. The smaller the value of the loss function, the more beneficial the control program is.

The final extension considered here is allowing the inflation control program to affect the Phillips curve to capture Lipsey's [24] empirical finding. In that study, Lipsey finds the inflation control program flattens the short-run Phillips curve. Possen [32] alters his Phillips curve equation during the control program because people become aware of the lower actual inflation rate, lowering the rate of growth in wage demands, and raising the level of output. In our models, the control program changes the expectations formation equation (and the LM function in Model 2) but not the Phillips curve, Eq (2.9). The Phillips curve could be altered by the same expression which represents people's belief in the control program in the expectations formation equation, Eq (2.8). Therefore, the Phillips curve would be of the form:

$$\ell n\ y = \ell n\ \nu + \gamma_1(\pi-\pi^e) + \gamma_2(\pi_g-\pi); \quad \gamma_1 > 0, \quad \gamma_2 \geq 0 .$$

Here, the term involving the controlled inflation rate would capture people's awareness of the actual inflation rate relative to the controlled rate. As the inflation rate falls toward the controlled rate, this expression causes the level of output to rise.

Now, the inflation control program directly affects the level of output through the Phillips curve. Since this extension does not permanently alter the equation, the post-control equilibrium would not be affected. However, the short-run time paths and the control equilibrium would differ from those obtained in this thesis. This

discussion is merely a brief look at the endless possibilities of expanding the analysis of the present inflation control program models.

## D. A FINAL WORD

This thesis has provided us with some insight into the effectiveness of a voluntary inflation control program. Even though the models analyzed are very simplified versions of an economy, the results obtained for the level of output and the inflation rate agree with the results of Possen's complex model [32]. Also the movement of the real interest rate is consistent with the short-run time path explained by Friedman [15]. Based on this, Models 1 and 2 provide a good description of the effects an inflation control program has on an economy. Unlike the control models reviewed in Chapter 1, Models 1 and 2 describe the short-run movement of the interest rates, and the expected and actual inflation rates. Finally, this thesis examines the stable and unstable post-control periods. The control models reviewed in Chapter 1 briefly examine the stable post-control period, but not the unstable case. Apart from providing realistic information about the effects of a control program, the models are also easily adaptable to changes and extensions for further analysis of the subject.

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APPENDIX I

MODEL 1

## A. SHORT-RUN DYNAMICS

We differentiate Eq (2.5) and Eq (2.6) with respect to time and eliminate the growth rate of output, $\frac{\dot{y}}{y}$, in these two equations and solve for $\frac{\dot{M}}{M}$.

$$\frac{\dot{M}}{M} = \pi - \mu\dot{\rho} - \lambda\dot{r} \quad . \tag{2.12}$$

Eliminating $\pi^e$ in Eq (2.8) with Eq (2.7):

$$\dot{r}-\dot{\rho} = (\beta-\delta)\pi + \beta\rho - \beta r + \delta\pi_g \quad . \tag{2.13}$$

Solving for $\dot{r}$ and $\dot{\rho}$ in terms of the other variables in these two equations:

$$\begin{bmatrix} -\lambda & -\mu \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \dot{r} \\ \dot{\rho} \end{bmatrix} = \begin{bmatrix} \frac{\dot{M}}{M} - \pi \\ (\beta-\delta)\pi-\beta r+\beta\rho+\delta\pi_g \end{bmatrix}$$

$$\dot{r} = \frac{1}{\lambda+\mu} \left[-\frac{\dot{M}}{M} + \pi + \mu\beta(\pi-r+\rho)+\mu\delta(\pi_g-\pi)\right]$$

$$\dot{\rho} = \frac{1}{\lambda+\mu} \left[-\lambda\beta(\pi-r+\rho) - \lambda\delta(\pi_g-\pi) + \pi - \frac{\dot{M}}{M}\right] \quad .$$

We replace $\pi^e$ with Eq (2.7) in Eq (2.9):

$$\ell n\ y = \ell n\ \nu + \gamma(\pi-r+\rho) \quad .$$

Recall that the long-run natural output level is $\nu$ and $\rho_\nu$ is the

corresponding natural real interest rate. Hence, Eq (2.5) becomes: $\ell n\ \nu = \alpha - \mu\rho_\nu$. Replacing this expression of $\ell n\ \nu$ in the equation above and solving for $\pi$ yields:

$$\pi = \frac{\mu}{\rho}\rho_\nu + r - (1 + \frac{\mu}{\gamma})\rho \quad .$$

Using this expression to eliminate $\pi$ from the equations involving $\dot{r}$ and $\dot{\rho}$ :

$$\dot{r} = \frac{1}{\lambda+\mu}\ [(1-\mu\delta)r - (1 - \mu\delta + \frac{\mu}{\gamma}(1+\mu(\beta-\delta)))\rho + \frac{\mu}{\gamma}(1+\mu(\beta-\delta))\rho_\nu - \frac{\dot{M}}{M} + \mu\delta\pi_g\ ] \tag{2.15}$$

$$\dot{\rho} = \frac{1}{\lambda+\mu}\ [(1+\lambda\delta)r - (1 + \lambda\delta + \frac{\mu}{\gamma}(1-\lambda(\beta-\delta)))\rho + \frac{\mu}{\gamma}(1-\lambda(\beta-\delta))\rho_\nu - \frac{\dot{M}}{M} - \lambda\delta\pi_g\ ] \ . \tag{2.16}$$

## B. EXAMINATION OF THE STABILITY CONDITIONS

Using the Routh-Hurwitz theorem, the necessary and sufficient conditions for stability in the case of two simultaneous differential equations are:

1. determinant is positive, and
2. trace is negative.

1. The Determinant

$$\begin{vmatrix} \frac{1}{\lambda+\mu}(1-\mu\delta) & -\frac{1}{\lambda+\mu}\left(1-\mu\delta+\frac{\mu}{\gamma}(1+\mu(\beta-\delta))\right) \\ \frac{1}{\lambda+\mu}(1+\lambda\delta) & -\frac{1}{\lambda+\mu}\left(1+\lambda\delta+\frac{\mu}{\gamma}(1-\lambda(\beta-\delta))\right) \end{vmatrix} = \frac{\beta\mu}{\gamma(\mu+\lambda)} > 0 .$$

This is automatically satisfied, for all the parameters are positive.

2. The Trace

$$\frac{1-\mu\delta}{\lambda+\mu} + \frac{-[(1+\lambda\delta) + \frac{\mu}{\gamma}(1-\lambda(\beta-\delta))]}{\lambda+\mu}$$

$$= \frac{-1}{\lambda+\mu}\left[\delta(\mu+\lambda) + \frac{\mu}{\gamma}(1-\lambda(\beta-\delta))\right] .$$

For the trace to be negative the following condition is required:

$$\delta(\mu+\lambda) + \frac{\mu}{\gamma}(1-\lambda(\beta-\delta)) > 0 .$$

This is the necessary and sufficient condition for stability.

C. WHEN $1-\mu\delta = 0$

Examining the slope of the $\dot{r} = 0$ curve shows it is vertical, $\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} = \infty$. Therefore, only two possible cases exist.

a. $$\left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} > 0 \; .$$

The stable case.

b. $$\left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} < 0 \; .$$

The unstable case.

APPENDIX II

MODEL 2

## A. SHORT-RUN DYNAMICS

We differentiate Eq (2.5) and Eq (3.2) with respect to time and eliminate the growth rate of output, $\frac{\dot{y}}{y}$, in these two equations and solve for $\frac{\dot{M}}{M}$.

$$\frac{\dot{M}}{M} = (1+\varepsilon)\pi - \mu\dot{\rho} - \lambda\dot{r} - \varepsilon\pi_g \ .$$

Eliminate $\pi^e$ in Eq (2.8) with Eq (2.7):

$$\dot{r}-\dot{\rho} = (\beta-\delta)\pi - \beta r + \beta\rho + \delta\pi_g \ .$$

Solve for $\dot{r}$ and $\dot{\rho}$ in terms of the other variables in these two equations:

$$\begin{bmatrix} -\lambda & -\mu \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \dot{r} \\ \dot{\rho} \end{bmatrix} = \begin{bmatrix} \frac{\dot{M}}{M} - (1+\varepsilon)\pi + \varepsilon\pi_g \\ (\beta-\delta)\pi - \beta r + \beta\rho + \delta\pi_g \end{bmatrix}$$

$$\dot{r} = \frac{1}{\lambda+\mu} \left[ -\frac{\dot{M}}{M} + (1+\varepsilon+\mu(\beta-\delta))\pi + \mu\beta(\rho - r) + (\mu\delta-\varepsilon)\pi_g \right]$$

$$\dot{\rho} = \frac{1}{\lambda+\mu} \left[ -\frac{\dot{M}}{M} + (1+\varepsilon-\lambda(\beta-\delta))\pi + \lambda\beta(r-\rho) - (\lambda\delta+\varepsilon)\pi_g \right] \ .$$

From Appendix I-A we have the equation for $\pi$,

$$\pi = \frac{\mu}{\gamma}\rho_\nu + r - (1 + \frac{\mu}{\gamma})\rho \ .$$

Using this expression to eliminate $\pi$ from the two above equations involving $\dot{r}$ and $\dot{\rho}$ :

$$\dot{r} = \frac{1}{\lambda+\mu}\Big[(1+\varepsilon-\mu\delta)r - \{1 + \varepsilon - \mu\delta + \frac{\mu}{\gamma}(1+\varepsilon+\mu(\beta-\delta))\}\rho$$

$$+ \frac{\mu}{\gamma}(1+\varepsilon+\mu(\beta-\delta))\rho_\nu - \frac{\dot{M}}{M} + (\mu\delta-\varepsilon)\pi_g\Big] \qquad (3.3)$$

$$\dot{\rho} = \frac{1}{\lambda+\mu}\Big[(1+\varepsilon+\lambda\delta)r - \{1 + \varepsilon + \lambda\delta + \frac{\mu}{\gamma}(1+\varepsilon-\lambda(\beta-\delta))\}\rho$$

$$+ \frac{\mu}{\gamma}(1+\varepsilon-\lambda(\beta-\delta))\rho_\nu - \frac{\dot{M}}{M} - (\lambda\delta+\varepsilon)\pi_g\Big] . \qquad (3.4)$$

## B. EXAMINATION OF THE STABILITY CONDITIONS

Explanations of the derivation of these conditions are the same as those presented in Appendix I-B.

### 1. The Determinant

$$\begin{vmatrix} \frac{1}{\lambda+\mu}(1+\varepsilon-\mu\delta) & -\frac{1}{\lambda+\mu}(1+\varepsilon-\mu\delta + \frac{\mu}{\gamma}(1+\varepsilon+\mu(\beta-\delta))) \\ \frac{1}{\lambda+\mu}(1+\varepsilon+\lambda\delta) & -\frac{1}{\lambda+\mu}(1+\varepsilon+\lambda\delta + \frac{\mu}{\gamma}(1+\varepsilon-\lambda(\beta-\delta))) \end{vmatrix}$$

$$= \frac{\mu\beta(1+\varepsilon)}{\gamma(\lambda+\mu)} > 0 .$$

This is automatically satisfied for all positive parameters.

2. <u>The Trace</u>

$$\frac{1+\varepsilon-\mu\delta}{\lambda+\mu} - \frac{(1+\varepsilon+\lambda\delta+\frac{\mu}{\gamma}(1+\varepsilon-\lambda(\beta-\delta)))}{\lambda+\mu}$$

$$= -\frac{1}{\lambda+\mu}\left[\delta(\mu+\lambda)+\frac{\mu}{\gamma}(1+\varepsilon-\lambda(\beta-\delta))\right] .$$

For the trace to be negative the following condition is required:

$$\delta(\mu+\lambda)+\frac{\mu}{\gamma}(1+\varepsilon-\lambda(\beta-\delta)) > 0 .$$

This condition is a necessary and sufficient condition for stability.

C. WHEN $1+\varepsilon-\mu\delta = 0$ $(\varepsilon-\mu\delta = -1)$

Examining the slope of the $\dot{r} = 0$ curve shows that it is vertical, $\left.\frac{dr}{d\rho}\right|_{\dot{r}=0} = \infty$. Therefore only two possible cases exist.

a. $\left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} > 0$ .

The stable case.

b. $\left.\frac{dr}{d\rho}\right|_{\dot{\rho}=0} < 0$ .

The unstable case.